The subject of ordinary differential equations is an old and beautiful one. Indeed, one of the first and most important differential equations expresses Newton's Second Law: F=ma. Of course in the three hundred years since this early beginning in classical mechanics, differential equations have been used to model an increasingly broad and rich selection of inherently dynamic phenomena. Unfortunately, a vast number of our introductory courses fail to convey this dynamic nature. Rather they are viewed as boring "techniques" courses in which assorted formulas for solutions are matched to a variety of seemingly unrelated equations. Students leave the course with the misleading impression that all ordinary differential equations have explicit solutions, when in fact quite the opposite is true. The numerical and qualitative analysis of equations receives little attention, particularly the latter topic. This inattention is regrettable, since in many "real-world" modeling situations, the most important questions often center on: how a solution changes in response to various parameters that appear in the equation? or how does the solution behave with respect to its initial conditions? We would be remiss in our educational mission, if we did not afford our students exposure to this manner of analysis.

Recently, however, Hüseyin Koçak [2] published PHASER, a remarkable piece of software that holds great potential to make this type of analysis, particularly the qualitative theory, more accessible to our students. The package produces sophisticated graphical displays of solutions to differential and difference equations and for a detailed description of the many features of PHASER, we cite the excellent review article of Bridger [1]. Here we simply note the features that make the package so attractive for the classroom. All of the equations that are built into the software appear in a very general format, allowing for parameter values to be changed and multiple initial conditions to be considered. This also applies to any "external" equation that a user might enter in for study. A variety of graphical displays (including 3-D images with perspective) is also available, and different numerical approximation schemes can be used to compute the solutions, so that comparisons of algorithmic efficiency and accuracy can be made. Since the package is entirely menu-driven and no programming skills are required, it is particularly well suited to serve as a "laboratory" environment for student self-discovery and exploration. For the instructor, there is a utility that allows an entire sequence of screen images to be saved to a file so that a "slide show" demonstrating a key concept or idea can be displayed in class. Note, however, that each screen image is actually redrawn at the time of display. Hence students can watch an individual solution evolve in time, as well as watching a sequence of such solutions, resulting from a sequence of changes in the underlying equation. In this way PHASER animates what are normally perceived as inanimate models.

At the University of New Hampshire we have been working to incorporate PHASER into our introductory sophomore level course since the fall of 1986. Support for this work has come primarily from the University's computer-aided instructional initiative, Project DISCovery, or Directions in Scholarly Computing. We have used three vehicles for this effort: large lecture demonstrations, student performance of specific experiments or "computer labs" designed to explore concepts, and individual self-instruction. Since the course is organized around large (125+ students) lecture sections, supplemented by twice weekly recitation sections, most of our efforts have involved the development of sets of in-class demonstrations. These have been displayed by using a number of large-screen projection systems. In the two parts of the project involving "hands on" use of PHASER, exercises are intended to be a paradigm for part of an applied mathematician's work. A computer experiment is performed and the resulting data (both numerical and graphical) suggest certain patterns. These form the basis of a conjecture, which must then be verified through rigorous mathematical analysis. Here we have had to rely on a self-selecting group of students each semester to work on the special "computer laboratory" assignments. Before discussing the natural issues of implementation that are raised here, we note that in the fall of 1989 we will be offering a special small section of ordinary differential equations in which PHASER will be used from the very beginning of the course and will in effect serve as the "text". For a more in depth description of both the in-class demonstrations and the "laboratory" assignments, see Zia [3].

Regardless of the software package being used, some type of projection system is absolutely crucial for in-class demonstrations, unless the class size is so small as to permit everyone to see a monitor image without strain. Our experience has been that a permanently mounted system complete with the appropriate computer hardware is the ideal situation, particularly for a large lecture section, as anything short of that poses an obstacle to the less technologically inclined instructor. A number of portable systems which use an LED display board with a standard overhead projector are also available and these can prove effective for the smaller class sizes. Since PHASER produces color displays, a color projector is recommended but not essential. Another logistical issue involves providing students access to machines. While the large demonstrations are useful, they are still only a passive learning experience. Ideally we would like to see an entire class actively engaged in the use of PHASER. Certainly as more micro-computer clusters are created and as more students obtain their own machines, we can move in this direction. In the interim we have designated some of our recitation sections as "computer oriented" so as to give hands on experience with PHASER to perhaps not all students in the course, but at least a larger group than one obtained by self-selection. If this avenue is taken, we do recommend that computer exercises replace rather than overload what is normally required for a recitation, as this approach helps to maintain student motivation.

Beyond these logistical challenges there is the question of the impact that the use of PHASER has on the material in a standard introductory course. Since graphs of any dependent variable versus time can be displayed, PHASER is entirely suitable for the analysis of autonomous scalar equations, which is the typical starting point of most courses. But rather than considering higher order equations next as is frequently done, we found it useful to consider first order linear systems and develop the necessary linear algebraic concepts, thus setting the stage for using PHASER to illustrate the techniques of phase plane analysis for linear and non-linear systems. (These would of course include non-autonomous scalar equations). Having discussed first order systems of equations, we could then move directly to higher order scalar equations and recast them as first order systems in the usual way. We found this particularly useful in presenting the classical damped and forced harmonic oscillator.

Having advocated the use of PHASER to include the study of numerical and qualitative analysis in the introductory differential equations course, we must be prepared to omit an area that is normally discussed and we submit that the typical two to three week section on power series solutions can be dropped. While the basic idea of approximating a solution by an infinite linear combination of "simple" functions, namely powers of x, illustrates a powerful theme in mathematics, the tedious algebraic manipulations required to compute the coefficients in the typical expansion really only serve to obscure the picture. In fact, we suggest that if an instructor deemed it absolutely necessary to present the technique of power series expansions, the use of any symbolic manipulation package or calculator would be extremely appropriate for these computations and would streamline the entire presentation.

Finally, we note that the "responsible" use of the computer as a tool for understanding must be encouraged. One way that this goal can be accomplished is through the creation of "computer counterexamples" and PHASER easily provides a wonderful example. The user simply calls up the equation for the undamped and unforced harmonic oscillator which in theory produces periodic solutions that look like concentric circles about the origin in the phase plane. But if the standard Euler method (which is not "self-correcting") is used to compute the numerical solutions rather than the default Runge-Kutta algorithm, the numerical orbits do not close up and in fact they spiral away from the origin! Thus an experiment with the simplest two-dimensional linear system, if performed without care, can lead to seriously erroneous data.

References

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