COMPUTER GRAPH ANALYSIS

Agnes Wieschenberg
John Jay College of Criminal Justice - CUNY

In a Pre-Calculus and Calculus course a considerable amount of class-time is spent on functions and the graphing of functions. To be familiar with the shape and characteristic properties of a graph means better understanding of the function that generated it. Therefore, the development of the visual imagery of our students can mean a great deal when it comes to grasping some of the fundamental concepts in Calculus.

New technologies, namely calculators and micro computers, that are available today at most Colleges and Universities can greatly enhance our struggle for achieving better understanding of the calculus material. As we use these new tools, however, some caution must be exercised since these sophisticated tools work for us in different ways than the earlier more primitive pencil and paper methods. One **must** be familiar with the "old" methods to know what questions to ask as we incorporate the new techniques.

In the spirit of the "Calculus for a New Century" movement, Professor Peter Shenkin and I have spent a great deal of time in developing software to be used in all Pre-Calculus and Calculus courses at the John Jay College of Criminal Justice. (Actually we developed software to be used by every required mathematics course at the College which includes courses in Algebra and Finite Mathematics.) Each class spends one week in our microcomputer laboratory, which is equipped with IBM XT and AT computers, for a hands-on computer experience. The software is "user friendly" so that students need no prior computer experience to use it. Modules include finding slopes and intercepts of linear equations, systems of linear equations, and quadratic equations. All of these modules offer algebraic solutions as well as graphs at the fingertip of the user. Other modules include Linear Programming (using the Simplex method), Probability, Statistics, Limits, Graphs and Integration using numerical methods. [1]

The focus of this paper is the use of the Graphs Module. The package will run on IBM PCs and compatibles (and is being converted to Macintosh capability), is an easy to use graphics package. The student may enter some pre-programmed functions that appear on the menu such as

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Exp(x) and Exp(2*x)
Log(x)
Sin(x) and Cos(x)
Sin(x) and Sin(x+1)
Sin(x), 2*sin(x) and Sin(2*x)
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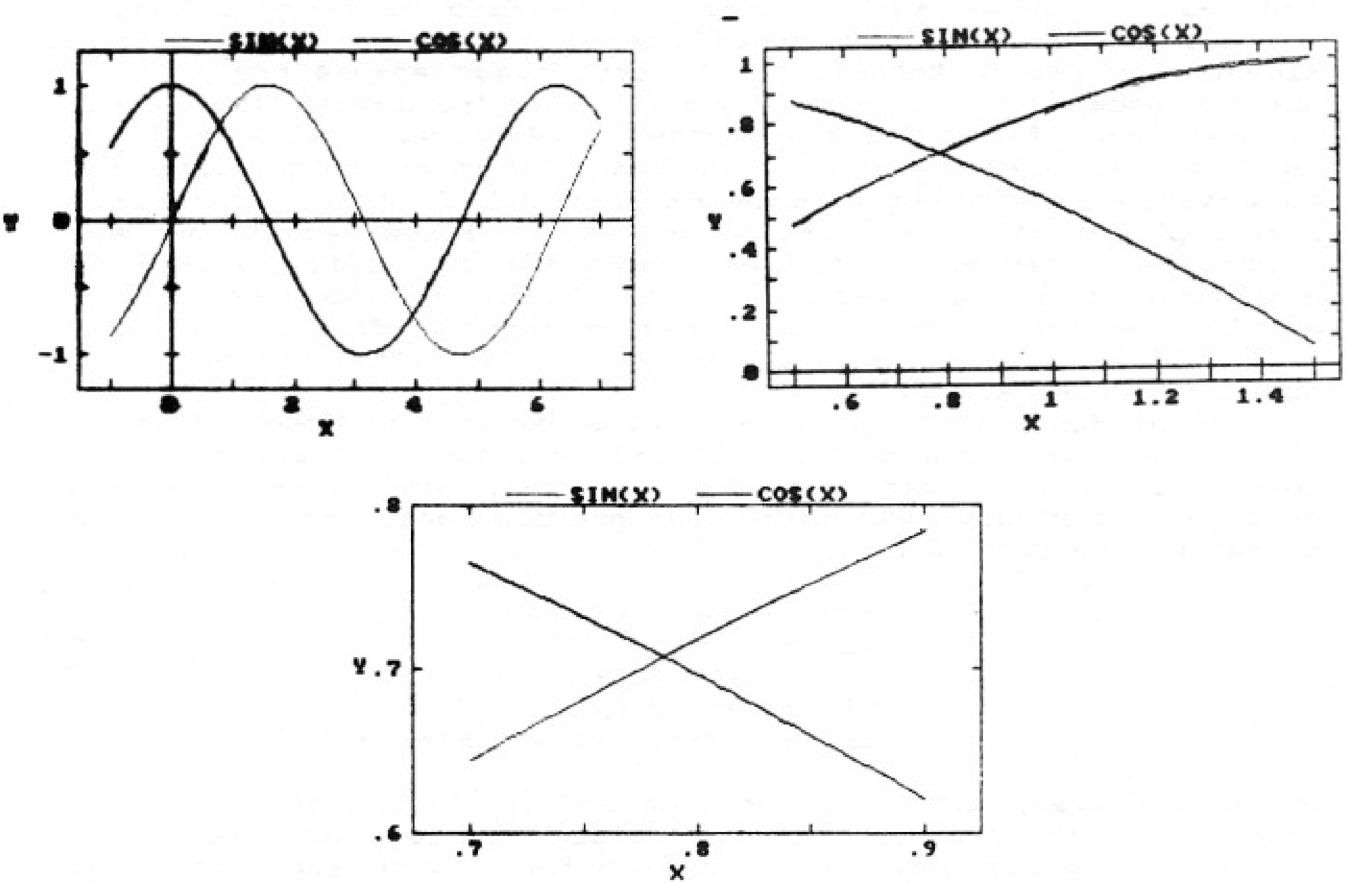
or interactively enter the function definition and the domain to graph. Both the function and the domain may be easily modified. The graph rapidly appears on the monitor. Upon examination of the graph, points of relative maxima and minima of the function as well as points of inflection and intervals of increase and decrease are easily estimated. The user can also enter two functions to be graphed on the same coordinate system and find intersections of the functions by estimating. The accuracy of this estimate depends on how far the user is willing to go to "close in" on the point of intersection.

The first example will illuminate the above points. Let us take the graph of two familiar functions, f(x)=sin(x) and g(x)=cos(x) and graph them on the same coordinates. We will try to estimate the value of y at the intersection of these functions in the interval between 0 and π. The questions that we may ask are: what does this point represent and how can we find an answer to three point accuracy? The answer to these questions may be simple to some but even this group will find it interesting to "zero in" on an estimated answer that can be more accurate than some trigonometric tables. Changing the parameters we can easily approach the desired number (in this case three decimal point accuracy). Now the student can check the tables and see that indeed the number that we found, .707 is corresponding to a 45° angle where the values of sine and cose are indeed the same. Thus the student learned not only the periodic nature of the sine and cosine graphs and that they have common points of intersections, but the nature of infinities between distances that otherwise appear finite. (Figure 1.)



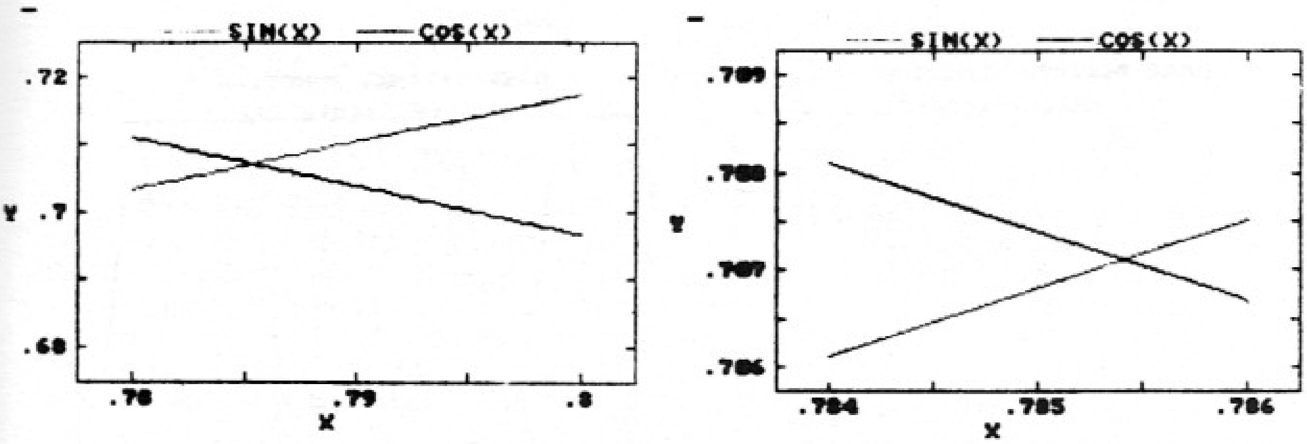
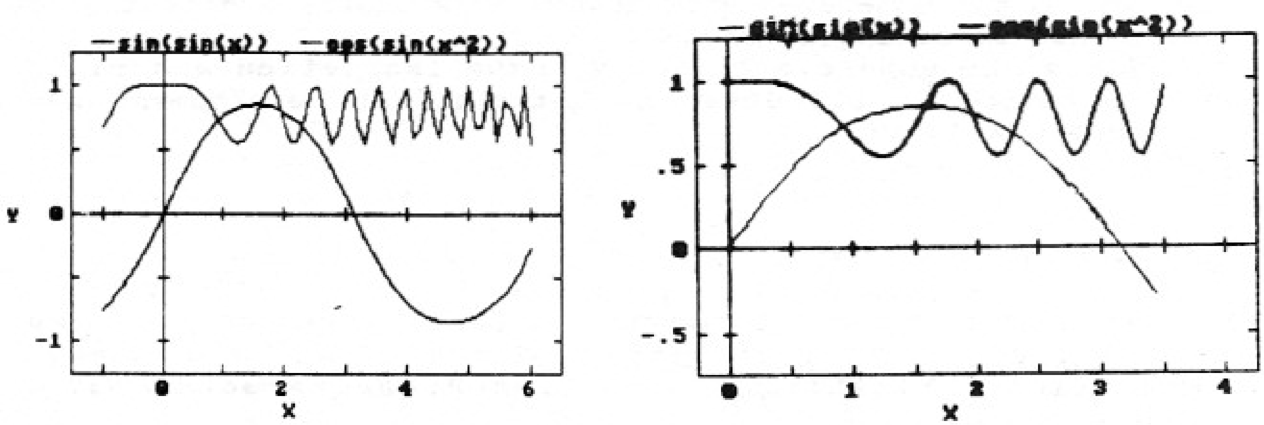


Figure 1.

Another lesson which may be learned is the fact that computations are very rapid on the computer but as we increase the difficulty level of the calculations, the computer will also need more time to produce an answer. Students may also learn from other examples as they experiment freely with the computer that computer errors can be significant in the calculation of certain formulas depending on the operations that are performed. Some students may investigate this further to develop ways to circumvent the problem.

Now let us can look at an example that is not so familiar. Let us select two functions: f(x)=sin(sin(x)) and g(x)=cos(sin(x^2)), and look at them at an interval from 0 to 6. After generating the graphs, all we have to do is to enter the functions and go from x=-1 to x=6, we immediately will see that there are four intersections in this interval. Using the above method we now can zero in on any one of those points and get an accurate estimate of its location. The same approach may be used for approximating maxima and minima, points of inflection and intervals of increase and decrease. (Figure 2)



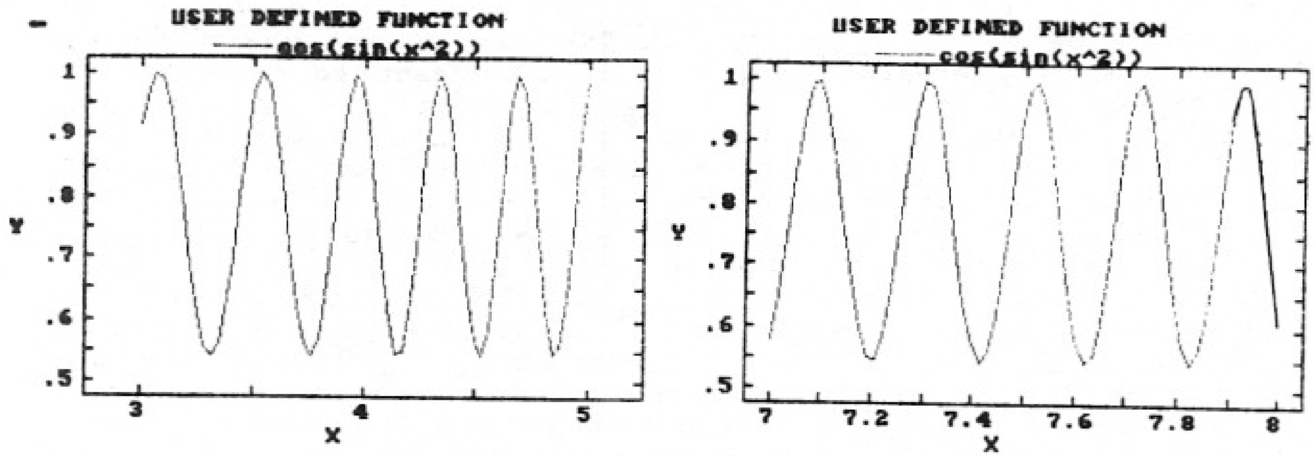


Figure 2.

If we want to investigate the graph of these functions as we are increasing the value of x, say maybe around x=30, we can change the parameters to values in this neighborhood. The user will soon discover that going from x=0 to x=30 will not result in a useful graph. To investigate how the the function cos(sin(x^2)) behaves as we increase the value of x, we must take smaller intervals for inspection. Since the range of the function is selected by the computer, some adjustments are sometimes needed in this selection. Thus easy experimentation becomes possible which in turn makes analyses accessible and more fun. Going on the wrong track will not result in frustration but rather learning of what steps will lead to fruitful results.

For functions such as f(x)=1/x, where there is a vertical asymptote, the computer will exhibit unusual behavior near the asymptote. The algorithm used in this module plots a value of zero in cases like this. The rest of the graph, however, is accurate.

For intervals where the value of the function is a complex number, the graph generated will behave irregularly indicating to the user that something is wrong.

Since the module allows free experimentation and rapid results students at all level of mathematical development can benefit greatly.

References

[1] Wieschenberg, Agnes and Shenkin, Peter. A Computer Companion For Undergraduate Mathematics Simon & Schuster Higher Education Publishing Group, Ginn Press, Needham Heights, Massachusetts, 1988.