Using the Computer in Freshman Calculus

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1. Introduction

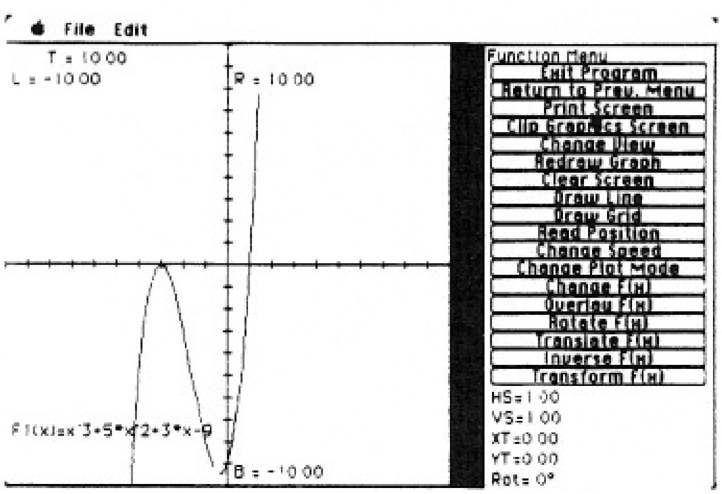
There are two separate and independent ideas addressed in this note. As a teacher of calculus I am interested in the manner in which computer/calculator technology can be used to help create a better learning model than that based on the traditional lecture. As chair of the Calculus Network in Michigan, a committee of the state section of the MAA, I am interested in the production of easily comprehensible textual materials that will assist the spread of this type of experimentation

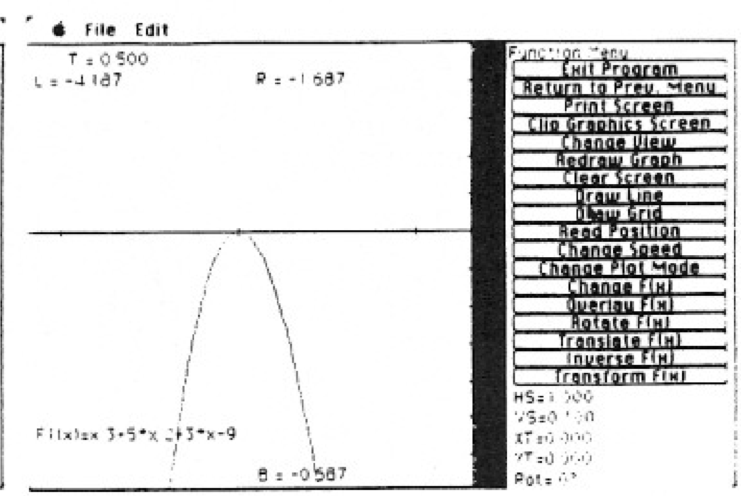
The class I am teaching with the assistance of computers is a first semester calculus class of 38 students, mostly first year students. Class is held in a computer laboratory two of the five days a week. The students work in pairs, each pair having access to a Mac Plus. During the course of the term, we have used Mastergrapher by Waites and Demana and True Basic Calculus by Kemeny and Kurtz. This note will concentrate on the first few days experience with Mastergrapher.

In the spirit of the curriculum suggested in the report of the Tulane meeting, Toward a Lean and Lively Calculus, I used the computer to introduce the basic functions of calculus: polynomials, rational functions, trigonometric functions, ln(x) and exp(x). While all but the last two might be thought of as review, the freshman student's actual knowledge of any of these functions and their graphs (not to mention the function concept in general) does not support such a position.

Methodology

The computer (or smart calculator) significantly assists the utilization of the best kind of contemporary learning methodology: one in which the students take an active role in learning through experimentation with the large number of graphs that the computer can construct carefully and quickly. The cubic pictured below on the left was my starting point. The careful examination of graphs is aided significantly by the "zoom - in" feature of this program (a feature common to most graphing programs). On the right we see closer picture of the interesting part of the graph, one from which the students can make a better approximation of values.





The following sample of the questions given in the written unit show the spirit of the experimentation being encouraged.

1. How many roots does F2 have and what are their x-coordinates?

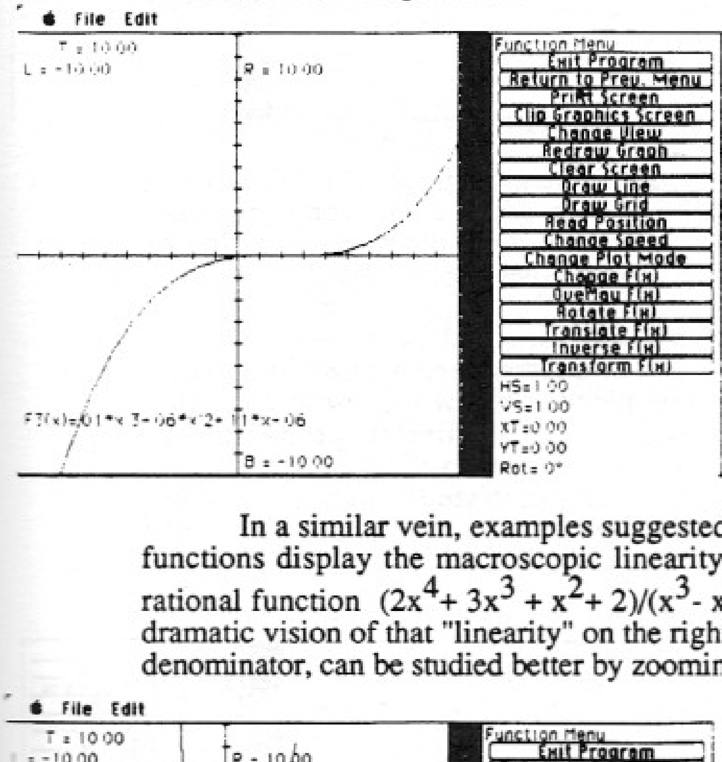
2. On the basis of your answer to Question 1, can you write a factored form of the polynomial?

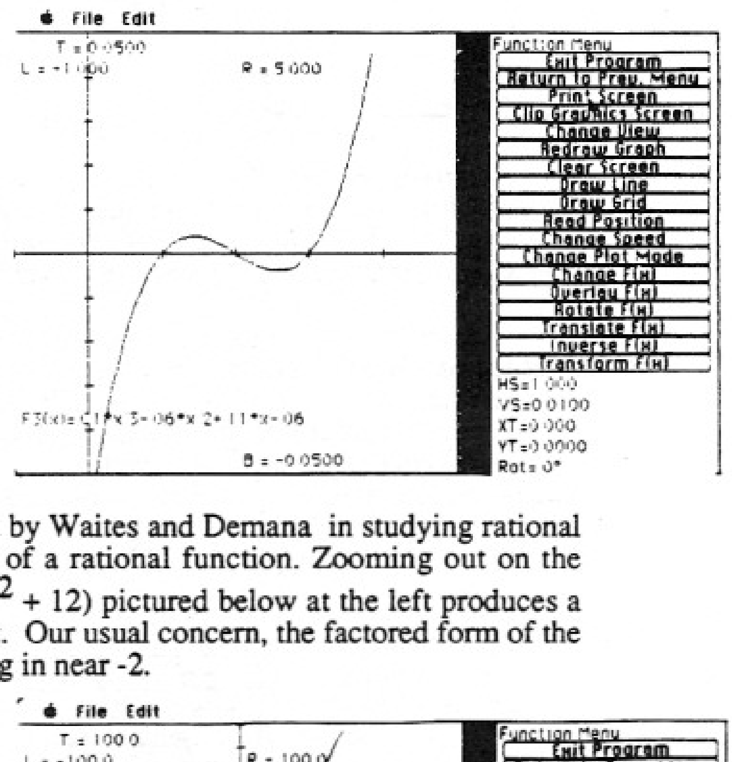
3. By changing only the constant coefficient of F2 can you create a cubic polynomial with 0,1or 3 roots? How or why not ?

4. By changing any of the coefficients, can you produce a cubic polynomial with the basic shape of F2, keeping the double root at -1, but raising the height of the relative minimum?

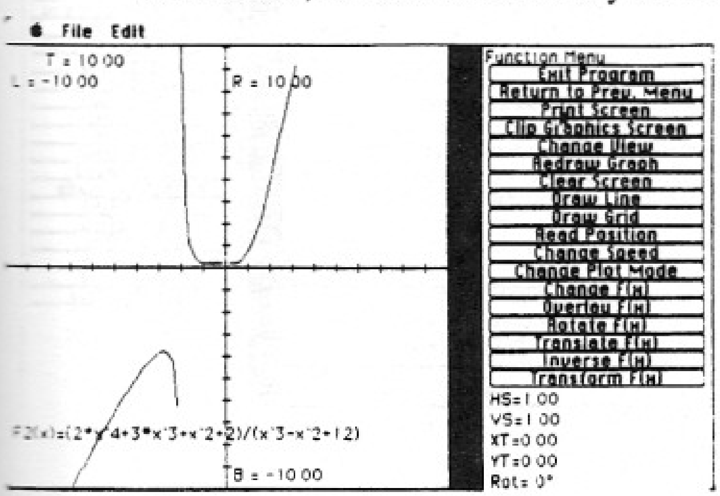
5. What are the possible shapes of cubics?

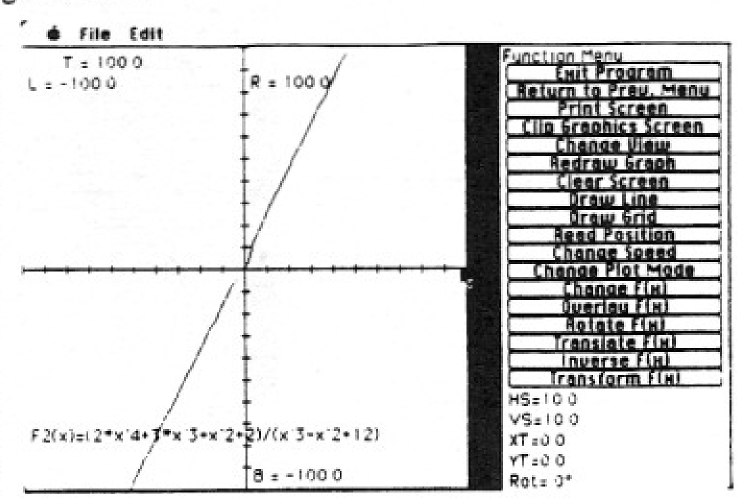
Cognitive dissonance, in the form of the surprising graph, becomes more feasible in the presence of machinery. The students are instructed to graph the cubic pictured in the graph on the left : a most unique cubic polynomial hugging the x-axis as it does for several units length. The students are asked to zoom until they determine what is going on. Two astutely made zooms with long squat rectangles produce the significantly scaled picture at the right. This is one of many good exercises about one of the principal themes of modern scientific investigation: the distinctly different information one obtains from the microscopic versus the macroscopic view.



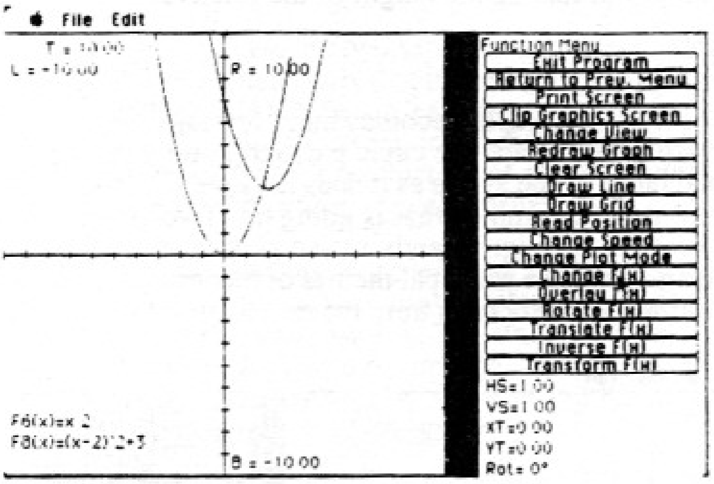


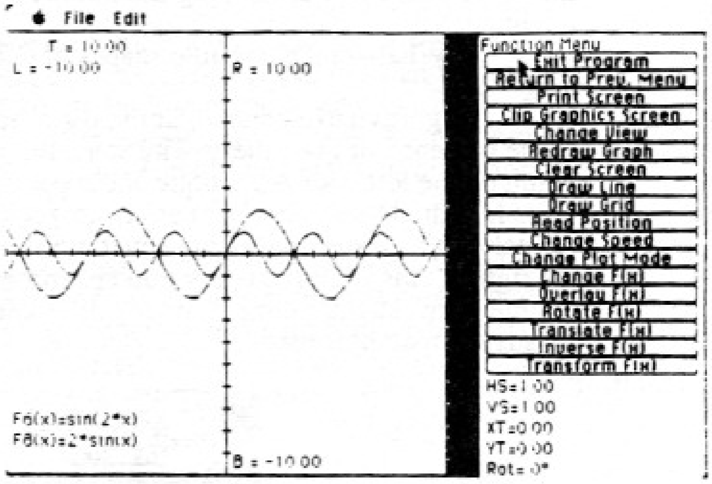
In a similar vein, examples suggested by Waites and Demana in studying rational functions display the macroscopic linearity of a rational function. Zooming out on the rational function $(2x^4 + 3x^3 + x^2 + 2)/(x^3 - x^2 + 12)$ pictured below at the left produces a dramatic vision of that "linearity" on the right. Our usual concern, the factored form of the denominator, can be studied better by zooming in near -2.





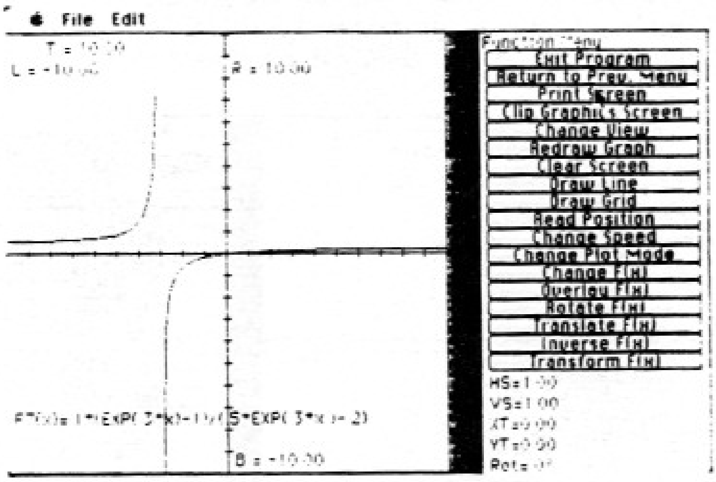
In a more organized fashion, the graphing units used with this program guide the student towards a constructive approach to graphing in general. Moreover, it gives the student a set of building blocks and processes from which graphs of many common functions can be produced easily and understood in a deeper qualitative sense. Translations, dilations, scalings, rotations and algebraic combinations of functions can be seen and practiced easily with computer. Space does not permit a more detailed presentation. Below are two examples that can be used to initiate the study.

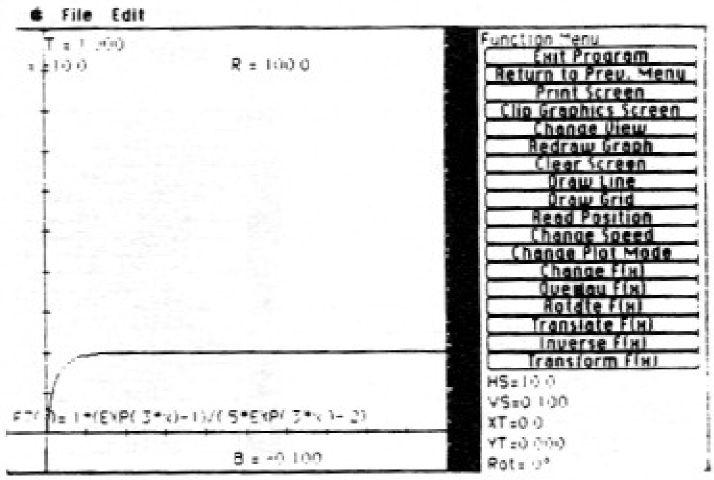




If this is thoroughly done, the students can be given the graph of a "black box function" f(x), for example, and they should be able to graph even such a general expression as 3 f(2x-4) + 1. This brings us a lot further along the road to graphical understanding than we could ever have hoped for without the new technology.

Before leaving the topic of graphing I was able to present to these students who had as yet learned little calculus, some applied problems of a rather deep nature and discuss their significance in graphical terms. The function pictured below represents the time change of a first order chemical reaction. They cannot solve the differential equation but they can interpret the graph. After a discussion of the chemistry to elicit the asymptotic value of the function, a scaling change of the x-axis to allow us to see 100 units and a scale change of the y-axis to focus on the small vertical values, produces a dramatic picture of what the chemistry tells you should happen.





Writing Materials

If productive experimentation with a technology assisted curriculum is going to spread, it is important to write materials that allow a teacher with little experience with computers to see the simplicity of such an approach and be attracted to try it.

Materials which integrate the computer operations in a step-by-step manner with the mathematics provide, I think, the best possibility of attracting large numbers of teachers to make the attempt. By "step-by-step manner", I mean a set of instructions which tells the teacher exactly which buttons need to be pressed and when. Moreover, this should be done in the course of presenting the curriculum.

The initial page of my set of graphing units is printed here. A similar set of "simple minded" instructions for zoming in and out, setting the window, using lines and other computer techniques has been integrated into the curriculum in a similar fashion.

GRAPHING FUNCTIONS USING MASTERGRAPHER

UNIT #1: Introduction to Graphing Functions

1. Developing Computer Procedure

There is an elementary sequence of steps which needs to be learned in order to make the Mastergrapher program easy to use. Once this sequence has been mastered it is easy to vary the procedure, introducing new techniques which make the program a powerful investigative tool. Perform the steps in the outline below.

1. Insert the Mastergrapher disk.
2. Double click on "Master Grapher 0.90".
3. Click anywhere inside the large box.
4. Select (Click) "Function Grapher" from the first menu.
5. Select (Click in circle) "Change or Remove Function" from next menu.
6. Select one or several functions from the function dictionary.

Select F1 as the first function we will investigate. If there is a black dot beside any other function click on it to erase it so only F1 is selected

7. Select "Previous Menu".
8. Select "Graph Function".

Notice that the cubic polynomial named as F1 is now graphed on a 20 by 20 unit grid centered at the origin. This grid can be changed in many ways. We will see this all later. There are values on this grid that are important for you to understand before going on. The values **T,B,R,** and **L** represent (respectively) "top", "bottom", "right" and "left". They give the dimensions of the grid which is represented on the screen. So this grid gives the set of points in $[-10,10]x[-10,10]$. In the lower right hand corner notice the values **HS** and **VS.** They represent (respectively) the horizontal and vertical distance between "ticks" on the horizontal and vertical axes. We will call this value the **tick value** . On this grid, then, each tick represents one unit value. The particular numerical values given for the variables in this picture are called the **default** values. When we change them later to do further investigations we will usually want to return to the default values at some point.

Do not perform the next step in the procedure since we want to get into the mathematics of graphing functions by asking some questions about the cubic polynomial which is in front of you. However, the last step completes the basic procedure so we list it here.

9. Click on "Return to Previous Menu" from the Function Menu to the right of the screen. This allows you to start the procedure again. Alternatively, click on "Change f(x)" to return immediately to the function dictionary.