Using the Group Experience Stack in a Modern Algebra Course

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A major obstacle to the introduction of extensive computer use into mathematics courses has been the skepticism of the instructor that the computer has a genuine role to play in helping students understand mathematical ideas. Software is available for doing calculations in many areas of mathematics, but in most mathematics courses, either calculation is not the focus of the course, or the instructor is unhappy that it is the focus. Thus courseware, while taking advantage of the computer's ability to calculate and display information, should also advance the more theoretical purposes of a course.

We have designed a Hypercard™ courseware stack, called Group Experience, for this purpose. Group Experience can perform a large variety of calculations, but it is a teaching tool, not an all-purpose group calculator. What makes this software into courseware is a set of problems that helps the student abstract principles from calculations performed on built-in and student-created examples.

The plan to emphasize the purpose of the course through the use of the computer extends from the design of the program itself to the design of documentation. Thus in addition to explaining how the package works, we describe to the student how the use of Group Experience fits into the learning process:

> Usually a problem relates to an example group that already appears with the problem, but some problems guide you in using the software to create the relevant group. In either case the problem will direct you to do computations and make observations about the example group that will lead you to more general conclusions about finite group theory. Completing the problem will often entail making a guess or conjecture about all finite groups; sometimes you will prove your conjecture as part of the problem, but in other cases that conjecture may be a theorem your teacher will prove in class. A general principle you learn in one problem will often help you analyze examples in later problems.

Thus nearly every problem consists of two parts, the first involving calculation and the second a conjecture and/or proof for the student to do without the computer. The Hypercard™ environment has helped in several ways. First, it is possible to incorporate the problem into the example, in the sense that the student looks at the multiplication table of a group and a problem relating to that group at the same time. (Fig. 1) Second, the convenience of creating a visual display in Hypercard™ permits the student to make his or her own examples relatively easily. (Fig. 2) Also, we are building in a system allowing a teacher to tailor the courseware to suit the course by adding problems to those included already.

The particular choice of Hypercard™ has had a substantial impact on the design of Group Experience, both in what became possible and what became impossible. Finite group theory was, as we expected, an excellent topic to deal with in the Hypercard™ environment. The existence of interesting example groups with small numbers of elements suggested presentation methods appropriate for the topic--and convenient in Hypercard™--that would not apply to most subjects. On the other hand, computations are relatively slow in Hypercard™, making elaborate calculation impractical.

An important component of any Hypercard™ courseware design is the format for examples, i.e. the form of screen image (called a *card* in Hypercard™) on which the example appears. For Group Experience, we chose a single format, the group multiplication table, for illustrating every example. We made this choice after some experimentation, including consideration of graphical representation of the elements of certain groups. Eventually, we opted for the simplicity of a uniform format, but that choice could easily have been different. A crucial task in designing Hypercard™ courseware for other subjects would be the choice of format.

The general design of Group Experience would be appropriate for many mathematical subjects. The ingredients are:

• a set of problems allowing the student to use the computer to do calculations, then requiring an abstraction of principles from those calculations;

• integration of each problem into an example illustrated on a card of a Hypercard™ stack;

• the capacity for the student to create and manipulate examples just like those already built in to the stack;

• the capacity for the teacher to tailor the courseware to a particular course by adding problems as desired.

We are currently using Group Experience in the junior-level modern algebra course at Franklin and Marshall, taught by Feldman with de Wit as Help Session leader. Getting the students started using Hypercard™ and Group Experience has been relatively simple. One class period devoted to demonstrating the software (and simultaneously introducing some new material) sufficed to teach the students how to use it. There have been a few minor glitches, but not enough to discourage the students or embarrass the authors.

It is difficult to assess the impact that Group Experience is having on the course, for there is no control group; the students in this course have never studied group theory before, so it is not clear how their ability to relate examples and theory might have been different if they hadn't been using the software. Group Experience seems to have generated no great excitement among them, nor has it seemed to engender any resentment. They have matter-of-factly accepted it as an integral part of the course.

From the instructors' point of view, Group Experience has worked well so far in that it has successfully directed the students in the abstraction of group theoretic principles from exploration of small groups; i.e., the software is working the way it should and the students are using it properly. The computer really does seem to be allowing the students to look at examples in more detail with less tedium, without depriving them of experience in calculation by hand. It is too early to tell the extent to which this experience is enhancing their understanding of group theory.

Figure 1

(Index) (Delete Card) (Quit HyperCard) (Print Table) (No Shadows)

Square

	0	90	180	270	H	V	D	D'
0	0	90	180	270	H	V	D	D'
90	90	180	270	0	D	D'	V	H
180	180	270	0	90	V	H	D'	D
270	270	0	90	180	D'	D	H	V
H	H	D'	V	D	0	180	270	90
V	V	D	H	D'	180	0	90	270
D	D	H	D'	V	90	270	0	180
D'	D'	V	D	H	270	90	180	0

sg. gen./check
nrml sg. gen./check
quit subgroup
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Conjugation

The Cayley table above is for the group of 8 symmetries of the square. Click on the elements 0, 180, H, and V listed along the top of the table. These elements should now be "shadowed"--it looks like you're seeing double when you look at them. Click the button labeled

(hide problem) (other problem) (print)

Figure 2

(Index) (Quit HyperCard) (Print Card)

Listing the Elements of a Group

1. How many elements does the group have? Type a number 12 or less, then click OK. 6 OK

2. Type in the group elements, starting with the identity. After each entry, hit the Tab key to advance the cursor.

3.

r0	r1	r2	f1
f2	f3		

4. Do you want to set up a multiplication table (Cayley Table) for this group?

○ Yes
○ No

Follow the instructions on this card to create the Cayley table for the 6 elements of the group of symmetries of the equilateral triangle. Give this group the name

(hide problem) (other problem) (print)