TESTING WITH TECHNOLOGY

Scientific calculators, programmable calculators, and graphing calculators are in the hands of our students. Sophisticated software is available for students' own computers and in campus computer labs. Society in general has been much more receptive of modern technology than we in collegiate mathematics education have been. It is unlikely that technology will go away (even if we ignore it); therefore, we must take a leadership role in integrating calculators and computers into the classroom.

My philosophy on the role of technology in teaching mathematics (primarily precalculus and calculus) has evolved over the past several years as I have observed students struggling to make sense of the power they find in their hands. I believe that we must first teach students how to use calculators and mathematics software, then we must instruct them in the use of this technology for mathematics exploration and problem solving. Moreover, I believe that to achieve this objective, we must test to this objective.

Contrary to the belief of many educators and laymen, the use of calculators does not diminish the need for students to think or to acquire basic skills. Successful use of calculators and "toolkit software" requires a higher level of understanding than that required for rote computation or template problem solving. As an added benefit, the use of electronic tools in math classes allows the instructor to make a very strong case for the need to "look back" (Polya's last step in problem solving). "Does my answer make sense?" becomes ever more important, as the slip of a finger has the power to change the magnitude of an answer.

How can calculators and computers be used in student assessment? Simply allowing students to use calculators on traditional tests is not the answer. We need to make technology a viable component of the curriculum. We need to instruct our students in the use of calculators and computers and test them on their basic calculator and computer skills and on their ability to use calculators and computers to do mathematics.

The necessary skills for using technology effectively range from a basic knowledge of the operation of the calculator or program being used to a thorough understanding of the underlying mathematics. Students frequently have problems performing "chain calculations" on calculators, especially if they require the use of calculator memory. Issues involving round-off error and overflow gain importance as students use calculators for routine computations. Interpreting answers becomes vital. Finally, students must learn to recognize when an answer is unreasonable, either as a result of incorrect keyboarding or faulty logic.

A selection of calculator exercises on exams and quizzes stresses the importance of these skills to students. Problems can be approached from either of two perspectives: students can be requested to do specific calculations or they can be asked to interpret calculator results presented by the instructor. In the first case, apparently simple expressions such as sin(85pi/4), e^e, and $5{,}872{,}113^{37}$ can pose seemingly insurmountable problems on most standard scientific calculators; as such, they are a challenge to students' understanding of the mathematical concepts involved. In the second situation, questions such as "Why does the graph of this function appear to be just three vertical lines?" or "Why does my calculator show sin(pi/2) = .0274···?" will lead students to discover the need to carefully analyze their procedures and results.

Accepting calculators and computers as standard equipment in the mathematics classroom necessitates a change in emphasis and approach for several topics. Techniques of integration, for example, become less important as calculators can do numerical integration with only a few keystrokes and computer algebra systems have the capability to do complicated symbolic integration. A topic that has previously received little emphasis in beginning calculus courses, but that now merits consideration, is error analysis. Important for the successful use of numerical techniques, error analysis has become feasible for beginning calculus students with the availability of sophisticated calculators and computer algebra systems. For example, the error bound for Simpson's Rule can be easily determined using a combination of computer algebra and graphics software.

Graphing calculators have the greatest potential of all of the technological innovations to impact on the way we teach precalculus and calculus. The ability to experiment with the graphs of functions enables students to gain a fuller understanding of the relationship between functions and their graphs, which results in a more meaningful concept of the functions themselves. This easy access to graphs aids in the understanding of such topics as absolute value, solving inequalities, tangent lines (hence derivatives), and limits. In addition, graphics calculators complement the study of curve sketching - generating a reasonable graph on a calculator frequently requires the use of graphing techniques from calculus along with the use of several screens for a complete picture.

The availability of graphs at the touch of a few buttons arguably has the greatest impact on the way we test in precalculus and calculus. "Sketch the graph of $y = (x-2)^2$" is not a challenging problem for students equipped with graphing calculators. However, the concept of transformations of graphs can be tested using a generic $f(x)$ and asking for the graph of $f(x-2)$, etc. Graphing techniques can be tested by requesting students to "Sketch the graph of a function having these properties . . ." or by supplying information about the function and its derivatives and asking for the graph.

Another effective use of technology for student assessment is found in the introduction of professional software (much of which is available in student editions), such as MathCAD and TK! Solver. In addition, this gives students a realistic look at how mathematics is used in the "real world" early in their studies. Projects using computer algebra systems and scratchpad software allow students to combine a variety of mathematical tools and techniques to solve non-trivial problems.

We teach best by example. If we expect our students to be proficient in the use of technology, we must be proficient ourselves. An excellent opportunity to use calculators and computers as our students use them is in the construction of exams. The most obvious example is the use of computers to generate graphs for exam questions. Furthermore, graphics capabilities make it possible to find the right function for a test question without having to search through dozens of textbooks.

Computers can also be used to randomly generate exams. There are commercially available test data banks and construction kits, as well as systems for computer generated and administered tests. For small classes, however, individualized questions can add variety to exams. A simple instructor-written program can randomly generate a different problem for each student in the class. Personal data can also be used to individualize questions. Personalized questions are especially suitable when a graphing or toolkit program is going to be used for the solution. Grading such questions often requires less time than is needed to give partial credit on a more traditional exam, as was the case when students were assigned randomly generated systems of equations to be solved using TK! Solver.

A technological revolution is happening now. As mathematicians, we have the opportunity to be at the forefront of this revolution. As educators, we find traditional values and methods being questioned. By combining the roles of mathematician and educator, we can utilize the available technology to enhance the teaching and testing of mathematics.

Gloria S. Dion
Department of Mathematics
Penn State Ogontz
Abington, PA 19001

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