INTEGRATING GRAPHING CALCULATORS, PC SOFTWARE, AND SYMBOL MANIPULATORS INTO A CALCULUS LAB

Robert Decker

John Williams

University of Hartford, Mathematics Department

West Hartford, CT 06117

Two sections each of Calculus I and II are being taught this year at the University of Hartford using programmable graphing calculators (Casio FX7000G), calculus software (EPIC), and a symbolic manipulator (muMath). The Calculus I part of the experiment will be discussed here. One hour out of four each week is spent in a computer laboratory, where 19 AT&T networked personal computers are available; each student is required to purchase the Casio calculator. Each week a laboratory assignment is given, which the students start during the in-class lab hour and finish out of class if necessary. The labs emphasize realistic problem solving, discovery and exploration; some of these labs will be discussed below. The calculators are used for in-class work, for homework problems, and can be used to finish lab assignments at home. We cover the same syllabus as the other four standard calculus sections.

There are several things which we accomplish using the lab format. One is to get students to attack realistic problems. Since they have muMath for finding derivatives, we can freely use any of the standard functions at a very early point in the course. We generally cover topics in the labs before they are introduced in the lecture. Thus the students get a chance to think about a topic before dealing with it formally. Sometimes this leads to discovering a property which is to be introduced later, and helps to cut down on lecture time. We also want to encourage group work; in the lab, there are two students per computer. The lab partners must interact with each other to get the lab done; watching the students trade and discuss ideas is a joy. Another goal is to encourage geometrical thinking, in addition to algebraic thinking. Finally, we require lab reports which demonstrate good communication skills. These must be written in correct English with each result explained in words.

The starting point for Lab I is a simple physics experiment. A mass is attached to a thick cord and suspended from the ceiling; the mass hangs at the level of the blackboard. The mass is displaced 20 inches from the rest position and released; every fifth return of the mass a chalk mark is made on the board representing the extreme position of the mass, and the corresponding time is recorded with a stopwatch. Due to the thickness of the cord, the motion of the mass is damped sufficiently so that the chalk marks end up being a few inches apart. The students are told that for small oscillations, a good model for the displacement y from the rest position as a function of time t is

$$y = A\, e^{-kt} \cos(wt).$$

The assignment is to adjust the parameters A, k, w to fit the data. This is accomplished in the lab using EPIC, although it can be done with the calculator almost as easily.

This lab gets the students to understand the difference between a variable and a parameter, and to discover the effect of each parameter on the shape of the graph. It introduces them to the experimental approach to mathematics; we want them to realize that there is more to mathematics than manipulating symbols. Each group comes up with slightly different values for the parameters, which leads to interesting follow-up discussions. For instance, is it better to fit the first two points, the first and the last or some other combination? What compromises should be made in trying to get a good fit?

Many of the other labs use the function developed in the first one. In Lab 2 the students are required to find the time values and velocities for the first time that the mass passes through the rest position ($y = 0$), and the first time the mass passes through the $y = 15$ position. To do this, they must solve an equation by successively enlarging the graph until the root can be determined to 4 decimal places. Then, the instantaneous velocity is estimated at these points using nearby time values and a difference quotient. This is done before the derivative is defined, but after an in-class small group session on estimating slopes of curved lines on a calculator. By graphing successive secant lines (performed by EPIC), one sees these lines approaching the tangent line. In a later lab, the velocity is found using muMath; this is still long before logs, exponentials and trig functions are formally introduced.

Lab 5 allows the students to discover the relationship between the graph of a function and the graph of its derivative. A polynomial is investigated first; both the function and its derivative are graphed together. The max's and min's can be approximated using the point locator of EPIC, and the roots of the derivative can be found using the root finder, whereby the above relationship is discovered. The student is then asked to apply this knowledge to the pendulum function and find its maximums and minumums on a small interval. This can be done with EPIC without using the formula for the derivative. It can also be done by taking the derivative with muMath and using the calculator to find the roots (we have given them a program for using the secant method to find roots on their calculators; see below).

Another lab involves solving equations with a combination of graphing and the secant method. In this lab, which is done on their calculators, we first give the students a program for solving equations using the secant method (we see this as somewhat preferable to Newton's method, since the derivative does not have to be programmed). Then they must find all roots of a given equation in a given interval. First a graph gives them approximate solutions, then the accuracy is improved using the secant method. Finally they are asked to find the last time value for which the pendulum from Lab 1 was 15 inches from the rest position. The graph must be magnified several times in order to find a good starting point for the secant method, since the last two roots of the equation they must solve are very close together.

Other labs involve discovering the product rule using muMath, finding areas through a sequence of better and better approximations, related rates problems using muMath, numerical-graphical exploration of differential equations (a supplementary program written by the instructors is needed here), and a discovery approach to the Fundamental Theorem of Calculus [1].

In addition to being available for use on the labs, the calculators are used on homework problems and in-class work. Besides the secant method for solving equations mentioned above, the students are given a program for Simpson's method for definite integration. These tools combined with the graphical capability of the calculators leaves the students with a powerful tool for dealing with problems not only in calculus, but in many other courses. One result is that the amount of time needed to cover the sections on max's and min's, curve sketching, and area finding is considerably cut down.

One effect of teaching calculus this way is the increase in active learning on the part of the students. They are continually challenged to come up with their own ways of solving problems. Sometimes what they come up with is not what was intended by the instructors; this is something we must learn to accept, and even encourage. For instance, after learning how to take derivatives, a student used finite difference quotients on one lab to find the equation of the tangent. This can be used as an opportunity to discuss exact versus approximate answers.

An advantage to using muMath is that equations with parameters can be used, such as the one used by the students in Lab 1. If particular values of the parameters are chosen, numerical experiments can be done. But if one wants to know what happens for any parameter values, muMath can be used to get certain information. Students come away with a better understanding of parameters, and the limitations of numerical approaches. Functions with parameters are rarely discussed in the standard Calculus I course.

The lab reports provide an alternative means of evaluation for the instructor. Although the students work cooperatively during lab time, each student must write their own lab report, where they have to assimilate and communicate the lessons of the lab. The instructor must emphasize this aspect in order to get good quality reports; math students are not used to using words to describe math problems.

We believe that all three technology components are important and each has a unique role to play. At this time there is no piece of software that has the graphics, numerical and symbolic capabilities needed, that is easy to use and that is available on PC's. Without the calculators, the students have nothing to use at home on homework problems, or to finish labs with. A scaled-down version of the lab approach could be used with the calculators alone.

The students gain an appreciation for the purpose of both calculus and the new technology by working on problems that would be hard to solve otherwise. Working with a function as complicated as the one used in Lab 1 without the technology would be difficult. Finding something like the velocity of the swinging pendulum would be hard to do experimentally without sophisticated equipment, hence the importance of calculus.

Reference

[1] Decker, Robert. "Discovering Calculus," to appear in The Mathematics Teacher.