EPIC

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EPIC (Exploration Programs In Calculus) is a microcomputer software package that permits users to investigate mathematical concepts. It includes mathematical style function input, an extensive function plotting routine, a module on limits, a module on integration, and a versatile polar coordinate and parametric equation curve sketching module.

EPIC supports the use of parameters in functions and easily shows how the graph depends on these parameters. Users may estimate the location of zeros, critical points, and inflection points, draw tangent lines and secant lines, and draw first and second derivative curves. Limits and difference quotients may be examined; the area under a curve or between two curves may be approximated by Riemann sums, the trapezoidal rule, or Simpson's rule. Intersection points of polar or parametric curves can be approximated and these curves can be "traced."

EPIC allows functions to be entered in multi-line textbook style, including restricted domain specifications and summations. Virtually any function in calculus texts can be entered and examined. EPIC can be used to enhance many topics in single variable calculus. Due to the required brevity of this paper, we will only discuss a few topics where EPIC may be useful in elementary calculus.

Continuity and Differentiability

EPIC supports the definition of functions involving more than one segment or section. For example, the function

$$f(x) = \begin{cases} \dfrac{x^2+1}{2x-3}, & x \le \pi \\ a\,\mathrm{Tan}^{-1}x + b\sin x, & x > \pi \end{cases}$$

is entered into EPIC as shown in Figure 1. The primary difference between textbook display of such functions and their input into EPIC is the inclusion of the double line to separate sections.

Many functions included in the discussion of continuity or differentiability in calculus texts involve multi-section functions and these functions may be copied into EPIC exactly as they are written in the text. This formulation versus a one line, computer-language definition enhances readability and student understanding, minimizes error in function input and enables the student to focus on the mathematics of the task rather than the translation of the formula to a computer-language expression.

Figure 1

Students could be assigned several problems related to functions of this type. For example, what values should be chosen for the parameters a and b to ensure that $f(x)$ is continuous and differentiable at $x = \pi$? The proper value of a is $a = (\pi^2 + 1)/((2\pi - 3)\arctan\pi)$. EPIC supports algebraic expressions as input for numerical information, so the student could enter this value as (π^2 + 1)/((2π − 3)arctanπ).

EPIC will draw tangent lines and secant lines; the secant line feature may be used to illustrate that, when the choices for a and b differ from their proper values, the curve essentially has two distinct "near" tangent lines at $x = \pi$, and hence is not differentiable there.

Inverse Functions

The graph of $y = f(x)$ is the collection of points $\{(x,y) \mid y = f(x)\}$. The inverse *relation* corresponding to f is the set $\{(y,x) \mid y = f(x)\}$. EPIC will draw this "inverse" curve and frequently it will be a multi-valued function. A single-valued function may be obtained by restricting the domain of the original function. Students can be asked to suitably restrict the domain of several functions as an introduction to the restriction of the inverse trigonometric functions to their principle values. For example, the graph of $y = f(x) = x^3 - 3x + 1$ and its inverse relation are shown in Figure 2. Since the max/min occurs at $x = \pm 1$, one possible domain restriction is $-1 \le x \le 1$. This restriction to f and its inverse are shown in Figure 3.

Figure 2

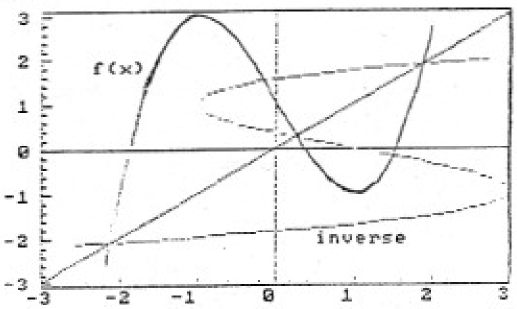
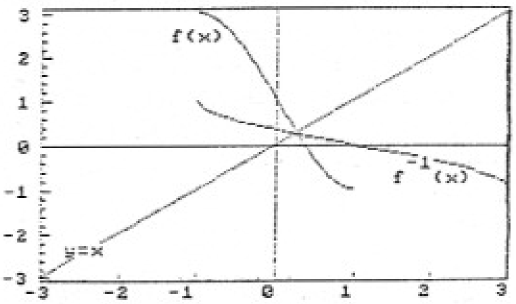


Figure 3



Relative Extrema and Inflection Points

Superimposed on the graph of f, EPIC will draw the graphs of f' and f''; the first and second derivatives may be computed by formula or approximated by difference quotients, at the user's discretion. EPIC will also find the zeros of f, f' and f'' (by Newton's method). The connection between a critical point and a relative extrema can be easily demonstrated as well as the distinction between *possible* inflection points and inflection points. In addition to drawing several graphs of a function by varying parameters in the function definition, EPIC allows two functions to entered and plotted at once, and then additional functions may be entered and plotted without destroying the screen. Thus many functions may be displayed at the same time.

The Computer May Mislead

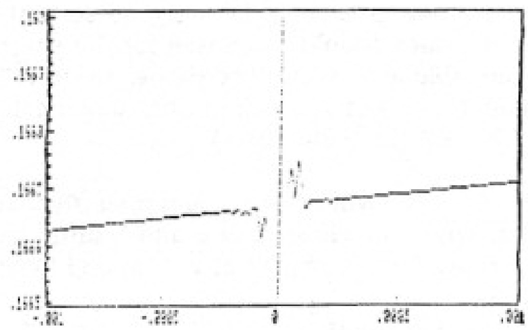
EPIC plots functions by evaluating the function at equally-spaced points throughout an interval selected by the user. (EPIC starts with 128 sampled points, but the user may vary this from 2 to 600.) Because of this sampling, important behavior of a function may be missed. An extreme example is the graph of $f(x) = \cos 256\pi x$ on the interval [0,1]. Sampling at $x_i = i/128$ yields a value of 1 for each i. Consequently, the graph appears to be a straight line at $y = 1$.

Another example is the graph of

$$f(x) = \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3},$$

for $x \neq 0$ and $f(0) = 1/6$. The graph of f on a small interval about the origin produces inaccurate function values due to round-off error. The graph is "bumpy" when it should be monotone and nearly constant. See Figure 4.

Figure 4

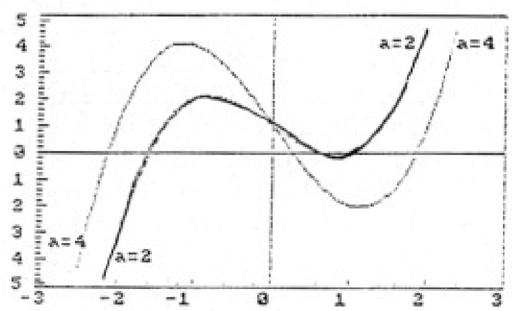


Implicit Differentiation

Consider the graph of the polynomial $f(x) = x^3 - ax + 1$. Figure 5 shows the curve obtained for $a = 2$ and $a = 4$. Increasing a seems to have moved the two leftmost roots left while the largest root has moved to the right. If we view $x^3 - ax + 1 = 0$ as an equation defining x as a function of a, then implicit differentiation yields

$$x'(a) = \frac{x(a)}{3x^2(a) - a}.$$

Figure 5



For the roots near -2 or 0.3, a quick calculation verifies that $x'(a) < 0$; for the root near 1.5, $x'(a) > 0$. This supports the observed movement of the roots. However, note that for the case $a = 2$, the function almost has a double root. For values of a smaller than that needed for a double root, there are no positive roots, leading to questions of when is a function $x(a)$ defined by an implicit relation. In this setting, EPIC may be used to motivate the need for more analysis of the situation.

Series

EPIC will plot functions defined by a series, including power series and Fourier series. Figure 6 shows the input screen for the series representation of Bessel functions, $J_n(x)$. Note the parameters in the upper limit of the summation and in the terms of the series. The first controls the number of terms computed to evaluate the function; the second parameter is the index of the Bessel function. Figure 7 shows the graphs obtained by choosing $m = 8$ and $n = 0, 1, 2$.

Figure 6

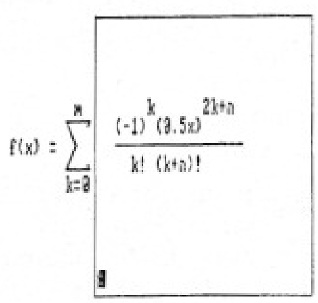
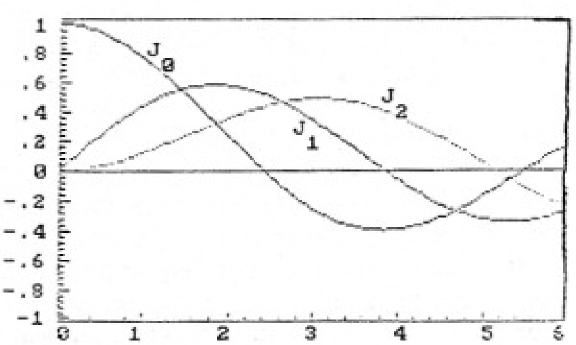


Figure 7



Additional Features

- EPIC enables the user to approximate area under a curve or between two curves by five methods: left, right and midpoint Riemann sums, the trapezoidal rule and Simpson's rule.

- EPIC contains a module that demonstrates the concept of limits. A small "window" (chosen by the user) of the graph of the function is magnified repeatedly, displaying details of the behavior of the graph.

- EPIC contains a module on polar and parametric equations. Some of the standard polar and parametric curves may be selected from a menu, or the user may enter the functions of his choice. A frequent question related to these curves is the precise order in which the curves are traced as the polar angle or the parameter varies. EPIC provides the answer to this question by slowly tracing these curves, with the tracing rate under user control.

- EPIC contains a calculator which "understands" most standard mathematical notation. The calculator may be invoked within every module. Parameters may be assigned to calculated values and then used in subsequent calculations. Up to ten formulas or calculations may be saved and recalled for later use.