Christine Browning

The Computer/Calculator Precalculus (C²PC) Project and Levels of Graphical Understanding

At The Ohio State University, a project group called the Calculator and Computer Precalculus (C^2PC) Project (Demana &Waits, 1988; Osborne & Foley, 1988) is moving toward increasing the attention given to graphing in the precalculus curriculum. The goal of the C^2PC project is to improve student understanding of functions and graphs by enhancing the precalculus curriculum through the use of computer and calculator based graphing. They are modifying the typical precalculus curriculum and applying computers and calculators to emphasize the correspondence between the numerical and algebraic representations of functions with their graphical counterpart. The number of examples, or the base for generalizing about functions and graphs, is increased by the use of computers/calculators. Students participating in the program, along with one precalculus class not in the C^2PC program that served as a control, provided the sample for this present investigation.

In order to determine the hypothesized increased understanding of functions and graphs provided by this modified curriculum, a measure of assessment needed to be developed specific to these areas of concern. Thus, the C^2PC project provided a motivation for the author to develop an instrument to assess student understanding of functions and graphs. The study (Browning, 1988) focused on developing a "Graphing Levels" instrument to determine if levels of understanding exist and can be characterized. The Graphing Levels instrument was then used to determine any growth in graphical "ability" in an attempt to assess any growth in understanding.

The title of the instrument indicates the author's belief that the understanding of functions and their graphs will occur in levels, perhaps hierarchical. This belief is based on research pertaining to the van Hiele levels of geometric understanding (van Hiele, 1958; Hoffer 1981, 1983; Usiskin 1982; Mayberry 1983; Fuys and Geddes 1984; Shaughnessy and Burger 1985, 1986; Crowley 1987). Although this research is based on levels of understanding related to geometric learning, a later work of van Hiele (1986) extended his ideas about levels to all mathematical learning. Very little research to date has focused on extending van Hiele's ideas to other areas.

Another major research project related to levels of understanding is the Concepts in Secondary Mathematics and Science (CSMS) program in England (Hart, 1980). Results of the CSMS study demonstrate the presence of levels of understanding in many mathematical topics (ratio and proportion, fractions, positive and negative numbers, graphs, vectors, algebra, etc). The research performed by the author was a "partial replication" of the CSMS project.

The research took place during Autumn Quarter, 1987 and Winter and Spring Quarters, 1988 at The Ohio State University, Columbus, Ohio. A 25 item instrument was designed, titled "Graphing Levels Test," and given to precalculus students at two city and two suburban high schools in central Ohio. The majority of the students were enrolled in the C^2PC precalculus classes with the remainder enrolled in a standard precalculus class. The Graphing Levels Test was used as both a pre- and posttest with only minor item modification on the posttest. Student interviews took place after both pre- and posttest administration. Pretest interviews were used as an aid in determining item modifications along with suggestions from mathematicians and mathematics educators. Results from posttest interviews aided in determining level characteristics.

Cluster analyses similar to that performed in the CSMS project was performed on pretest results from a random sample of 125 students from the original 211 to determine levels of graphical understanding. The group of 211 students was comprised of students from both groups in the original sample. Four clusters or levels were suggested from the analyses. The number of levels was based on the "facility band splits." A criteria span of not more than 20 percent was chosen, based on the CSMS project suggestions. A Guttman scalogram analysis was performed using the pretest random data to determine the validity of a hierarchy. A coefficient of reproducibility higher than 0.9 is considered to indicate a valid scale. The coefficient for the pretest random sample was 0.9975 indicating a hierarchy.

An examination of clustered items and review of posttest interviews suggested item characteristics for each level. The following is a sample of characteristics for each level.

Level 1: recognition of the graph of a parabola placed in different positions, simple interpretation of information from a graph, and development of initial vocabulary.

Level 2: translation of verbal information into a simple sketch of a graph, use of initial vocabulary learned in Level 1 and simple interpolation from a graph.

Level 3: usage of properties of graphs of functions to determine functions from non-functions, recognition of connection between a graph and its algebraic representation, and usage of properties of functions to construct graphs.

Level 4: usage of given information to construct a graph, usage of information from a graph to deduce more information, and recognition of what is required to find from given information.

As one advances from Level 1 to Level 4, it was noted the items became more complex, required more interpretation, required relating more ideas, depended on more knowledge, and required more complex problem solving strategies.

The pretest clusters were compared with posttest results from a split sample, that of the C^2PC group (n=125) and the control group (n=32). The level structure was essentially preserved for both groups, thus furthering the validity of the level construct.

Since the levels were found to be hierarchical in nature, a crosstabulation of the pretest data was done. Level scores were computed by alloting one point for each correct response to the items within each level produced by the cluster analysis. A criterion of correctly answering at least two-thirds of the items within each level was chosen as suggested by the CSMS project. The results of the crosstabulation showed the majority of students (77.5 percent) operating at Levels 0, 1 or 2 at the beginning of their precalculus year. A posttest crosstabulation showed the majority of the control group, 68.7 percent, remained at Levels 0, 1, or 2 while 73.1 percent of the C^2PC students were operating at the higher levels of 3 and 4. These results suggest the use of graphing technology within the precalculus classroom will provide for increased student understanding of graphs.

Another indication of growth in graphical understanding was found in comparing the posttest means of the five C^2PC classes with the one control class. All five posttest means were significantly higher ($p < 0.05$) than the control posttest mean of 13.7. The higher posttest means again provide evidence that the graphing technology makes a difference in the students' graphical understanding.

Results from this study indicate changes for the precalculus curriculum. Technology needs to be incorporated into the curriculum, specifically that related to graphing. If the precalculus curriculum is to prepare one for calculus, groundwork for calculus concepts and the corresponding skills must be laid. Part of this important groundwork is related to an understanding of functions and their graphs. As noted by the NCTM Commission on Standards (1987), among the most important of mathematical connections to be made in the curriculum are those between algebra and geometry. These connections can and should be developed in the precalculus curriculum. Graphing technology, as demonstrated by this study, is a definite aid to the student in developing the mathematical connections (see Pinker 1981, 1983 for further discussion on interpreting and making "connections" with visual displays). Technology provided C^2PC students with an increased example base and a greater number of graphing experiences than the control, furthering the C^2PC students' understanding of the connection between a function and its graph.

The hierarchical nature of the graphing levels suggests an order in the presentation of the topic . The greatest task for the educator is to allow students opportunities for making their own connections and not to provide shortcuts early on. For example, instead of telling the students what the graph of a quadratic equation looks like, allow them to experiment, a feat now possible with the aid of a graphing calculator. Not only does this allow the student to make their own generalization of what the graph of a quadratic looks like, it provides them exercises in creating quadratic equations.

In summary, results of this study show learning of functions and their graphs occurs in levels. The hierarchical nature of the levels implies an order to the presentation of topics related to graphing.

Results also imply the use of technology in the classroom improves student understanding of functions and their graphs by providing an increased example base. The interaction between student and computer also provided for making mathematical connections, connections necessary to understand and use graphs.

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