EFFECT OF COMPUTER GRAPHIC USE ON STUDENT UNDERSTANDING OF CALCULUS CONCEPTS

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The state of calculus course content and instruction has been called into question due to perceived student difficulties in understanding concepts and also due to the ready availability of computers and advanced calculators (c.f. Douglas, 1986; Steen, 1988). The literature concerning student difficulties in calculus suggests that students enter the course having a discrete concept of number. This concept of number is incompatible with the study of continuous phenomena (Confrey, 1980). A graphic representation, especially as displayed on a computer, holds promise for the development of understanding of the continuous processes and properties studied in calculus. Further, a perusal of the global features of functions represented by Cartesian coordinate graphs (Janvier, 1978) elicits further support for the use of graphs in the development of calculus concepts.

Based on the theoretical and empirical literature, student understanding of selected calculus concepts as developed through the use of a Cartesian coordinate graphical representation system were investigated. The purpose of the study was to compare the relative effectiveness of calculus instruction varying in the level of use of a graphic representation in developing the concepts of limit, continuity, and derivative on: (a) student facility with the use of a graphic representation in calculus; (b) student ability to solve applied, symbolic routine, and symbolic nonroutine calculus problems; (c) student attitudes toward mathematics; and (d) student attitudes toward the use and usefulness of graphs.

Subjects (N = 163) enrolled in four first-semester calculus sections at Western Michigan University participated in one of four treatment conditions: (1) Graphics (G), exposure to a computer-graphically developed conceptual course; (2) Graphics Plus (G+), exposure to the same course as G subjects plus provision of computer graphics software and related supplemental assignments; (3) Standard 1 (S1), exposure to a graphically-developed conceptual course without the computer; and (4) Standard 2 (S2), exposure to a traditional skill-oriented course.

In the experimental course presented to the G and G+ subjects, concepts were presented through real world applications which were modeled graphically, often through the use of computer graphs. The graphic results were then translated symbolically. A brief description of the development of the concept of derivative follows.

The Concept of Derivative as Developed Through Computer Graphics

The graphical development of the concept of derivative begins with a dynamic visual representation of the relationship between two quantities and their corresponding rate of change. Through the use of the program, SPIDER (Beckmann, 1988), students observe a "spider" climbing a wall as a graph of his position with respect to time is drawn on the screen. Students observe that a graph can represent the relationship between two quantities and that the shape of the graph informs them of how the quantities change with respect to each other.

The spider situation is followed by a discussion of average velocity from the point-of-view of a commuter versus that of a police officer observing the commuter. Average velocity is determined similarly by the commuter and the police officer, determining the change in distance divided by the change in time over which the travel took place. If position is written as a function of time, students suggest that an expression for average velocity is:

$$\text{Average velocity} = \frac{f(b) - f(a)}{b - a} \quad .$$

The commuter situation is modeled graphically. Students observe that the above expression is that of the slope of a secant line to the graph through the points (a,f(a)) and (b, f(b)). Asked to consider the police officer's perspective of this situation, students suggest that his interest is in instantaneous velocity at some time c in the interval (a,b). They further suggest that this is the limit of the above expression as the length of a time interval around c is reduced to zero. Asked to interpret instantaneous velocity graphically, students suggest that a secant would be drawn between two points which are very close together. They further suggest that the slope of such a line would better approximate the instantaneous rate of change as intervals of x became smaller.

Using *Master Grapher* (Waits & Demana, 1987), a variety of functions are investigated by graphing a function then magnifying the graph several times around a chosen point. Students observe that many continuous functions are locally straight on sufficiently small intervals. Asked to interpret the behavior of the displayed function with respect to a position vs. time situation, students suggest that the slope of the secant line containing the endpoints of the interval shown on the screen estimates the average velocity of the object whose position/time graph is displayed. When the graph appears to approximate a straight line, the average velocity closely approximates the instantaneous velocity over the interval displayed. If the graph of the function cannot be made straight over any interval containing a point (c, f(c)), then the instantaneous velocity cannot be determined at x=c.

Continuing the discussion, the program, SECANT (Beckmann, 1988), is used. SECANT draws successive secant lines through the points (c, f(c)) and ($c+h_i$, $f(c+h_i)$) where $h_i = h_{i-1}/2$. Students suggest that, as the length of the interval between c and $c+h_i$ decreases, the resulting secant line appears to intersect the graph in one point. Students recall that they have seen a line with such a property, a tangent, in previous work with circles. The definition of a tangent is generalized to include tangents to any curve.

The definition of the derivative is then given as follows:

If y = f(x), then the ***derivative of y with respect to x at x = c*** is

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} .$$

if this limit exists. When the limit does exist, it is called the ***instantaneous rate of change*** of y with respect to x at x = c, or graphically, the ***slope of the tangent*** to the graph of y = f(x) at x = c.

As described, the concept of derivative evolves from a discussion of a graph representing the rate of change of the quantities position vs. time. The rate at which these quantities change determines the shape of the graph of the function representing them. With respect to the commuter situation, the expression for average velocity is interpreted graphically as the slope of a secant to the curve. As the width of the interval over which a secant line is drawn decreases, the resulting secant line closely approximates the graph of the function. Globally, this secant appears to be the tangent to the curve at a given point. This development uses graphic and natural language representations in parallel to build meaning for the symbolic representation of the concept of derivative. It is representative of the approach taken throughout the course.

To analyze the data gathered in the study, two investigations were undertaken. In each investigation, prior calculus experience was used as a blocking variable for cognitive measures on two levels: (1) prior experience and (2) no prior experience. Covariates used in the analyses with cognitive variables were scores on a pretest of precalculus competencies and responses pertaining to attitudes toward mathematics.

In Investigation 1, comparisons were made between the G and G+ sections on : (a) performance on routine applied, routine symbolic, and nonroutine symbolic questions; (b) performance on the departmental final exam and its subscales; (c) changes in attitude toward mathematics; and (d) attitudes toward the course. Multivariate analyses of covariance were performed for the graphic cognitive variables. χ^2 tests were conducted for the affective variables. These analyses revealed no significant differences ($p < .05$) between the G and G+ sections on any of the cognitive or affective variables. Scores on cognitive measures for G+ subjects were higher than for G subjects, but the differences were not significant. Such findings indicate the need for further study. Attitudes pertaining to the use of graphs were overwhelmingly positive.

For Investigation 2, multivariate analyses of covariance were performed for cognitive variables. χ^2 and LSD tests were conducted to detect changes in the affective variables. Analyses of variance were conducted to compare student attitudes toward the course. Significant differences favored: (a) the S1 section over the G+ section on routine symbolic questions; (b) the G section over the S2 section on nonroutine symbolic questions; and (c) the S1 section over the G and G+ sections on the final exam questions pertaining to limit, continuity, and derivative. Questioning the validity of covariate measures for the standard sections in the analyses of (a) and (c), analyses of variance were performed on cognitive variables. A significant ($p < .05$) difference was detected on symbolic nonroutine questions favoring the G section over the S2 section. No other differences were detected. Student attitudes were generally positive, both toward the course and toward mathematics. Retention rates were found to be much higher in the conceptually-developed sections than for the technique-oriented section.

Results suggest that developing the calculus concepts through the use of a graphic representation system, especially through computer graphics, can positively affect student understanding and interest without necessarily negatively influencing skill acquisition.

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