The Effect of Technology on Teaching College Mathematics

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Both the candidates for President in the recent election spoke about becoming the "Education President." Neither, however, gave the slightest impression that he understands the problems facing education in the U.S. today and the dire consequences if we don't do something about it. If we don't, the best is certainly not yet to come.

There is much compelling evidence that we are failing to educate our youth to be effective citizens in a world increasingly dependent on science and technology. Moreover, the evidence is that things are getting worse. This evidence – which it is not my intention to detail here – consists of statistical data (such as the test scores of American children vis a vis their counterparts in other countries [2,4] and the declining enrollments in scientific and technological disciplines), an increasing body of anecdotal evidence about deficiencies in basic knowledge of American high school and college graduates, the widespread belief of university faculty that students are coming to college ever less prepared to engage in meaningful intellectual activity and mounting indications that American business and industry is, on average, ever less competitive in the world marketplace.

It would be nice to be able to say that technology offers an opportunity to improve the situation just described or even to turn it around. Unfortunately, it's not so. Deep societal and structural problems in the U.S. must be addressed if increasing educational mediocrity is not to result in scientific and technological mediocrity and then economic mediocrity – in short, in second class nationhood. What can be said, however, is that, while not sufficient, it is necessary that technology become widely used in the teaching process in American schools and colleges if we are going to see significant improvements in American education.

Some History

The United States became the world leader in science and technology in the middle of this century without any significant use at all of technology in teaching and virtually none at all in teaching mathematics. So why do we need it now? Very simply, because computers have changed the shape of knowledge and what subject matter it is important to teach in ways that demand the use of technology – computer technology, really – in teaching mathematics.

I had my first experience with using technology in a college course in the autumn of 1950 when I studied Numerical Analysis with Zdenek Kopal at M.I.T. Long hours over a hot Friden – or was it a Marchant? – filling up large ruled sheets with differences and related quantities in the quixotic search for solutions to ordinary differential equations was my introduction to technology used in teaching and to computing technology generally. Probably due to some (temporary?) softening of the brain, I loved it. But not so much that 10 years later when I started teaching numerical analysis, I jumped at the chance to assign students problems to solve on a real computer (an IBM 1620) even though I had to teach them a language (inevitably Fortran – sigh!) so that they could use the computer.

The single most remarkable thing about the use of technology in education is that today, almost 30 years later, essentially nothing has changed in the use of technology in education. An exaggeration? Yes, but not so much of one. Computers are used today in mathematics courses other than just numerical analysis such as linear algebra and differential equations. But they are not widely used even in courses like differential equations where their use should be routine. And they are seldom used in calculus or discrete mathematics courses where they are almost indispensable. And when computers are used at all, it is virtually always in the same old-fashioned way to give students assignments whose results will allow some insight into complex computational problems which cannot be obtained from pure analysis. Rarely – very, very rarely – are computers used in American college *classrooms* as a part – an essential part – of the teaching process. It is the thesis of this paper that much of undergraduate mathematics and almost all of lower division college mathematics cannot be taught properly without the use of computers in classrooms. Which is to say that we shortchange all of our students when we teach calculus or linear algebra or discrete mathematics without the use of computers in the classroom.

Computers in the Classroom

Before I discuss the value of computers in the classroom for teaching mathematics, let me dwell on the reasons why they are so seldom used in this way. I should note that my emphasis here is on computers not calculators although the latter, too, seldom make their way into college classrooms even though the new breed of symbolic calculators have an important role to play in college mathematics.

One reason for the absence of computers in classrooms is that, with few exceptions, American colleges and universities provide inadequate or nonexistent facilities for computer use in the classroom. It is *not* sufficient to provide micros which can be wheeled into classrooms on carts whose screens can then be projected by another portable device on a display screen which can be seen only with difficulty by most of the students in the classroom. What is needed are permanently installed computers, permanently installed projection equipment and overhead monitors or screens which allow good viewing from any seat in the room. Such facilities are not very expensive today [3]. But, nevertheless, most American colleges do not have a single classroom so equipped, and very few have more than one or two classrooms so equipped.

Hardware without software is, of course, of no value. How much software is available which, however user–friendly it may be outside the classroom, also provides instructors in classrooms with facilities which are easy and flexible to use? Not too much yet but there is some for teaching, say, calculus. Although the production of such software is difficult and challenging, it is quite certain that lots more good classroom software is on its way.

The chief barrier to the use of computers in the classroom is, however, neither hardware nor software problems. It is the inability of American college mathematicians to recognize the value of such facilities and their unwillingness to make the effort to use the facilities which are available. Because it is an effort. It is not sufficient to insert an occasional computer demonstration in the same calculus lecture you have been delivering for 20 years (although even this would be worth doing). What is needed is a total rethinking of what those lectures are intended to accomplish and, therefore, the development of, effectively, a new course. However, it may not be lectures that you want to give at all any more, at least not lectures in the classic mold. Instead, you should probably be running more interactive classes, perhaps dividing students into small groups while using the computer as the catalyst to developing understanding and problem–solving abilities. Hardest of all: You will have to replace teaching mechanical skills with teaching mathematics. All of this is *work* – hard work. It is hard to blame college faculty for their unwillingness to undertake such an effort when college administrations are seldom willing to recognize, with rewards or released time, the remaking of an old course in a new light.

There is also, I fear, a still widespread perception among college mathematicians of the computer as an essentially computational device. Despite the fact that computers have always been general symbol manipulators rather than "computers" and despite the development of symbolic mathematical systems (so–called, but wrongly called, computer algebra systems) such as MACSYMA^R, *Maple* and *Mathematica*™, the number of mathematics teachers who "think symbolically" when viewing computers is not large. This also needs to change if mathematics instructors are to perceive the opportunities to use computers in the classroom.

I must also mention graphics. Although all mathematics instructors believe that a picture is worth a thousand words and, indeed, regularly draw pictures on chalkboards or overhead projector slides, the potential of dynamic graphics – changing diagrams on a computer screen – has hardly been understood at all. Those who defend calculus as the bedrock of college mathematics usually do so on the basis of the importance of calculus as a mathematics of *change*. (Too often – and incorrectly – calculus is called *the* mathematics of change but no matter.) It is, therefore, ironic that virtually all teaching of calculus uses static tools – chalkboards and overhead projectors – in preference to a tool, the computer, that for the first time offers a dynamic medium for teaching dynamics.

So, if you don't start to teach mathematics with computers in the classroom you will miss major opportunities to enhance student learning and, as well, you will likely perpetuate a mind set that prevents you from making effective use of computers outside the classroom.

I now turn briefly to the use of computers in the classroom for the teaching of the most important courses in lower division mathematics:

Calculus
Discrete Mathematics
Linear Algebra
Differential Equations

Calculus

There is no portion of the current calculus curriculum whose teaching would not be enhanced by a computer in the classroom. The computational, symbolic and graphical facilities of such a system would – one or all – be useful every day in the calculus classroom. Wouldn't a system which allowed you to demonstrate dynamically the progression from secant to tangent aid almost all students in grasping the limit concept? If you still want to teach volumes of revolution, wouldn't it be a big help to see the volume forming and to picture the slices? Wouldn't the computational and graphical facilities of a computer calculus system be just what you need to show the connection between Riemann sums and integrals? And wouldn't a symbolic system designed for the classroom enable a more effective presentation of, say, the chain rule than you are now able to give?

My conclusion is that every period of a calculus class would be enhanced by a computer system designed for the classroom. Such systems are becoming available [1] although as yet they have generally not been designed with the aim of fully integrating the computer into the classroom. And, of course, we need textbooks which look at the computer as an integral part of the learning process not just an add on for those instructors who want some "enhancement" of their courses.

Discrete Mathematics

Ironically, although the movement to teach discrete mathematics to freshmen and sophomores has been motivated by the changes wrought in mathematics by the computer, there has been little use of computers in teaching discrete mathematics. This is particularly surprising because algorithms play an important role in many discrete mathematics courses. Probably the reason for the lack of use of computers in (or out) of the discrete mathematics classroom is the lack of software for teaching discrete mathematics. This is unfortunate but not surprising given that so many of the people most interested in teaching discrete mathematics seem to have been writing books rather than paying attention to other pedagogic matters.

But this should change fairly quickly in the next few years. Two examples of areas where good software for teaching discrete mathematics is obviously needed are:

- *Mathematical induction* which should play a major role in discrete mathematics courses but is often hard for students who have seldom, if ever, been exposed to proofs before. There are lots of ways that computer software could be helpful in teaching induction of which I mention only one. Induction is often taught in what I like to call the "mathematics as magic" mode in which students are presented with the result of a (usually algebraic) problem and then shown how to prove the result correct by induction. A much better approach is to use the *inductive paradigm* – compute, conjecture, prove – to infer the result and then prove it. For the "compute" portion of the paradigm, the computer is invaluable in presenting students with data in which to look for patterns.
- *Graph theory* whose essential visual nature makes it a natural for computer graphics; in addition, the many algorithms associated with graph theory can be derived and explained much better with a computer than without one.

Linear Algebra

If you believe, as I do, that a more algorithmic, computational approach to linear algebra than the classical abstract vector space approach is desirable – but even if you don't – computers have an important role to play in the linear algebra classroom. Systems like MATLAB, although designed more for out-of-classroom than in-classroom use, nevertheless suggest the kinds of things that could be done in a classroom to present a subject such as Gaussian elimination in a more dynamic and understandable fashion than is possible with chalkboard or overhead. But even abstract linear algebra has many topics for which a computer in the classroom would be very useful. For example, linear independence and basis would be easier to grasp using a symbolic–graphical system which allowed bases to be constructed and linear independence to be tested.

Differential Equations

It is my impression that differential equations textbooks of the past decade or two have usually contained a chapter on numerical methods. Such chapters are usually at the end of the book and are, I think, seldom taught. More recently some lovely computer systems for teaching differential equations, notably that developed by John Hubbard and Beverly West at Cornell University, have been developed although intended more for student use outside class than in-class themselves. But differential equations, not just in its computational aspects, generally needs consistent classroom support from a computer. Such bread-and-butter parts of differential equations courses as linear equations with constant coefficients need the support of a computer system to do the algebra and to present examples. Good computer software would also make it possible to show conveniently the close connections between differential equations and difference equations, something which is rarely done in undergraduate mathematics.

I have not even mentioned the impact of the computer – and the symbolic calculator – inside and outside the classroom on *what* you teach in the courses discussed above. Methods of integration is the canonical example of an important topic in calculus courses which needs to be rethought in light of the power of symbolic calculators. More generally, all topics in mathematics whose aim is the attainment of a symbolic or graphical skill can only be justified today if it can be reasonably argued that they aid the understanding necessary to deploy mathematics and to study further mathematics. This is a large topic, beyond my scope here, but one closely related to the use of computers in teaching mathematics.

Technology and Mathematics Pedagogy

In principle, you could use computers in the classroom on a continuous, daily basis without any essential change in your *philosophy* of teaching mathematics. You could still be a professor presenting knowledge to students in a fairly classical manner and giving them exercises and problems to be done outside of class, perhaps also using a computer. But to use the computer in teaching in this way is akin to what was done in the early days of business applications on computers when, essentially, previously used manual systems were emulated on a computer without giving any thought to whether or not the computer suggested new ways of doing business data processing. A computer is a foreign object in a classroom; properly understood, this means it is imperative to rethink the role of a professor teaching students.

I alluded to this earlier when mention was made of the need to consider interactive teaching and small group work. Here I want to discuss generally the impact of computers on the teaching of mathematics. In particular, I want to make a few remarks about *discovery* learning. It is a truism among teachers at all levels from kindergarten through graduate school that what you discover yourself you tend to know and understand better than what you are shown by others – your teachers. Still, it appears that with, perhaps, a very few notable exceptions like George Polya, teachers hardly make any attempt to have their students discover knowledge and, indeed, when they do make such an attempt, it is usually a failure. Do computers – in and out of the classroom – provide opportunities for discovery learning of a qualitatively different kind than any other educational technology – including books – have provided heretofore? I think so although candor requires me to admit that I can adduce no very useful evidence to support my contention. Still, some remarks may be useful in getting others to think more seriously on this matter than I have been able to yet.

Mathematics has been called the "science of patterns" [5]. Any such capsule description of a discipline is bound to be incomplete and even misleading. But it is true that doing mathematics at any level involves the search for and recognition of patterns in symbols, in data, in mathematical representations generally. Much of our teaching consists of defining these patterns for our students and then drilling them in applications of the patterns. How much better it would be if students could be led to recognize the patterns themselves so that the understanding they would achieve thereby would make most of the drill unnecessary and would enhance their ability to apply these patterns in unfamiliar situations.

In this context a computer may be said to be a pattern generator and a pattern manipulator. As in my earlier example of mathematical induction, computers can be used by mathematics instructors to generate and manipulate examples of general patterns in ways which can (may?) lead students to discover the generalizations for themselves. If we can learn to use computers this way, particularly in the classroom, it will surely revolutionize how we teach much – almost all – mathematics and it will surely greatly increase our effectiveness as teachers.

I have neither the space – nor the knowledge – to pursue this further here. But I do believe that all college teachers of mathematics should begin looking at the computer in the classroom as a pedagogic tool dramatically unlike any others which have ever been introduced into the classroom. The potential revolution mentioned above will only come after great effort, and it will probably come so slowly that it will look like evolution. Still, the real importance of symbolic systems and graphical systems is not what they do themselves but what they suggest about how mathematics should be taught generally.

A Final Remark

An immediate contribution that college mathematicians could make to the teaching of mathematics concerns their efforts in teaching prospective precollege teachers of mathematics. For both prospective secondary teachers of mathematics and all prospective elementary teachers there should be a course in the use of technology in the classroom for the teaching of mathematics (and, perhaps, for other subjects as well).

Students need to get used to and to become comfortable with technology in the classroom as early as possible. Not least among the effects of doing this will be that the next generation of college mathematics teachers will embrace technology far, far more rapidly than has the current generation.

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