

# TESTING, TEACHING AND TECHNOLOGY<sup>1</sup>

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In higher education mathematics is at an interesting juncture. We are in the midst of a shift in the environment for processing ideas and information. Technology is becoming a pervasive element of the context in which we operate as faculty in colleges and universities. Changes in environment tend to be irreversible. Short of major traumatic events, technology is here to stay as a feature of the environment in which we live and operate.

The challenge is simple: Do we use the technology in teaching mathematics, or do we Latinize the curriculum and our instruction? We can turn mathematics into a dead language. Alternatively, we can examine that environment to see:

- How can we capitalize on the positive characteristics?
- Are there actions we can take to smooth the transition to incorporating constructive use of technology in instruction?
- Are there features that change how we should design curriculum, instruction, and testing?

John Harvey (these proceedings) has made insightful comments concerning testing and teaching. I am electing to elaborate on some of the themes he has established partly to reinforce some of his wisdom and partly to examine some other features of what I perceive as critical in using technology in mathematics instruction. Ten years ago, I could not imagine myself making a presentation of this sort. I resented the computer. I saw it between me and the mathematics I wanted to do or to teach; a clumsy hurdle that I elected not to hurdle. I saw mathematics teachers in high schools teaching courses titled "Computer Algebra II" where the teachers spent a third to a half of the instructional time teaching programming rather than mathematics. I saw that in most instances students were encountering significantly less mathematics and learning considerably fewer ideas and skills.

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But the computer software and hardware has changed. Access to mathematics via technology is easy and direct. Menu driven technology and significantly greater power makes the mathematical ideas readily available. Indeed, I can readily shift to instructional methods that help students deal directly with problem solving, modeling, and generalization concerning a variety of mathematical ideas. The computer appeals to my basically conservative orientation to curriculum and instruction: I can feature mainline skills, understandings, problem solving and proof better with technology than without. I project this improving as the technology continues to evolve.

Following, I will identify some critical issues that we need to deal with in teaching and testing mathematics in a technology enhanced environment for mathematics instruction.

## Testing

Computers and calculators should be used throughout testing in mathematics. However, we need to enlarge testing and evaluation to encompass one feature of student performance seldom recognized as important in the environment for doing mathematics. We must begin to focus on whether students choose the most appropriate tool for a problem situation in mathematics. Indeed, we need to adjoin this type of behavior to our set of goals for mathematics instruction. Last week I observed a student in precalculus locating the zeros of a function. The function was quadratic and readily factorable. The student in this quiz situation elected to graph the function with a graphing utility. He made some very correct moves indicating he understood the mathematics of functions and graphs. He even dealt with the idea of error correctly. But, he had no decision tree to help him decide whether it was appropriate to work with paper-and-pencil or to use the computer graphing package. He wasted considerable time in changing the viewing rectangle and dealing with error in reading the graph; he should have simply factored the polynomial to obtain the zeros. He did not look at the exercise and make a decision about whether he should use the technology or not.

We must work to teach the judgment of which tool to use to do mathematics. Further, we must design testing alternatives that will examine whether or not students are attaining that judgment. I think that Harvey's comment about the fear of the technology turning students' heads to mush reflects, in part, that we have not recognized that we need to expand what we look for in examining mathematics behavior to encompass the choice of tool to be used for particular mathematics. The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (Commission on Standards for School Mathematics, 1989) argue that this is an important judgmental skill for every level of the curriculum. Figure 1 indicates the decision tree that the *Recommendations* advocate for every student to understand and employ. I think the decisions are important for students at the university level.

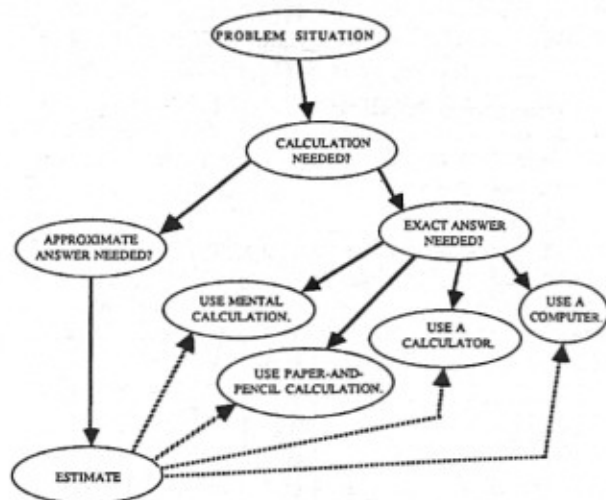


Figure 1. Decisions About How to Calculate

**What Do Test Items Measure?** An issue of prime concern is whether testing in the paper-and-pencil format does accommodate without significant adaptation to evaluation in a technological environment. We do not always know what students respond to or what understandings trigger correct responses.

Following are four items (Figures 2-5) we used in testing precalculus classes in the Calculator and Computer PreCalculus (C<sup>2</sup>PC) Project. The items are relatively standard fare for functions and graphing. The items were tested in two formats, one with a graph present and the other without the graph. The data given are the correctness rates on the items in the two formats. In some cases it appears the graph is a significant help; in others, not.

The first item appears on the surface to be of a type that the graph should help the correctness rate. In fact, the data indicate no such power obtains from the presence of the graph. One can argue that the typical student appears to summon the two-point algorithm and, if anything, the graph confuses. In an era when graphing utilities and calculators are readily available, do we know what we are testing?

In a Cartesian coordinate system, what is the equation of the straight line passing through point  $(0, -5)$  and parallel to the straight line whose equation is  $y = 2x + 3$ ?

- A.  $x + 2y + 5 = 0$
- B.  $2x - y - 5 = 0$
- C.  $2x + 3 = -5$
- D.  $2x - 5y + 3 = 0$
- E.  $2x + y + 5 = 0$

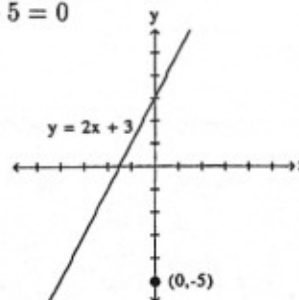


Figure 2. First C<sup>2</sup>PC Example Item

	Pretest	Posttest
Graph	53	74
No Graph	50	73

The second item appears to be moderately easier without the graph. Can we tell whether students think numerically or examine and interpret the graph? If a student enters this item on a graphing utility, it is the same as having the item given in the graphing format. Apart from the time it takes to enter the function on the utility, do we know what is triggering the students' responses?

If  $xy = 1$  and  $x$  is greater than 0, which of the following statements is true?

- When  $x$  is greater than 1,  $y$  is negative.
- When  $x$  is greater than 1,  $y$  is greater than 1.
- When  $x$  is less than 1,  $y$  is less than 1.
- As  $x$  increases,  $y$  increases.
- As  $x$  increases,  $y$  decreases.

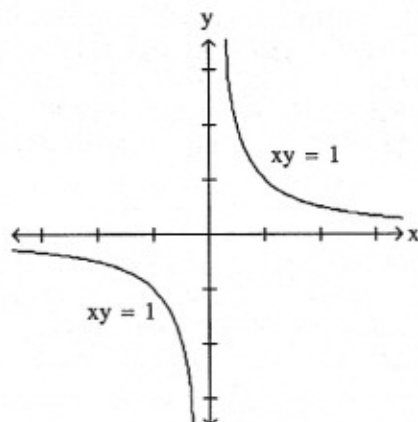


Figure 3. Second C<sup>2</sup> PC Example Item

	Pretest	Posttest
Graph	66	72
No Graph	61	78

The remaining two items have results that are in more of the predicted direction. The graph appears to help in producing correct answers. However, the use of a graphing utility completely rearranges the time demands of the item. Our judgments of how long an item will take go out the window with the use of a graphing utility. And we need to assure some sort of comparability of equipment for our students.

Which of the following,  $(x-1)$ ,  $(x-2)$ ,  $(x+2)$ ,  $(x-4)$ , are factors of  $x^3 - 4x^2 - x + 4$ ?

- Only  $(x-1)$
- Only  $(x-1)$  and  $(x+2)$
- Only  $(x-2)$  and  $(x+2)$
- Only  $(x+2)$  and  $(x-4)$
- Only  $(x-1)$  and  $(x+4)$

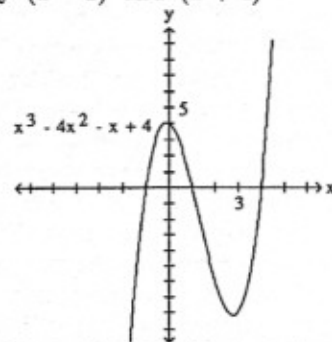


Figure 4. Third C<sup>2</sup> PC Example Item

	Pretest	Posttest
Graph	44	81
No Graph	35	54

The function  $f$ , defined by

$$f(x) = \frac{(x-1)(3x+1)}{(2x-1)(x-2)},$$

is negative for all  $x$  such that

- $-\frac{1}{3} < x < 3$
- $-\frac{1}{2} < x < 2$
- $1 < x < 3$
- $-\frac{1}{2} < x < 2$  or  $2 < x < 3$
- $-\frac{1}{3} < x < 3$  or  $1 < x < 2$

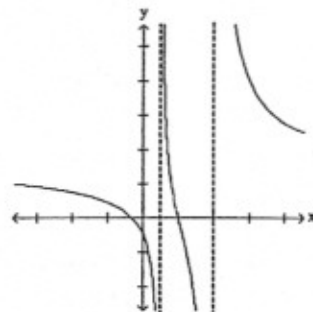


Figure 5. Fourth C<sup>2</sup> PC Example Item

	Pretest	Posttest
Graph	25	64
No Graph	23	39





testing and curriculum as well as providing different means of structuring tests. However, part and parcel to restructuring testing means that we must gain more information concerning item and response characteristics.

### Teaching

The use of technology will redirect your teaching and your thinking about curricular emphases. Decisions you make concerning content selection and treatment will change. I have visited a number of classrooms during the last two years that are using the technology enhanced curriculum of C<sup>2</sup> PC. The following comments concern the redirection of teaching and the decisions that seem obvious to me from my observations.

**Exploring, Experimenting and Problem Solving.** Use of graphing utilities encourages a more mathematical behavior on the part of students. Given a function, students can try things to see the effect of given parameters. One of our students in working with polar equations accepted the challenge of figuring out the effects of change of parameters in  $r = a \sin n\theta$  by experimenting with the choice of  $n$ . Most mathematicians have a good sense of what happens with different positive integers. Our student experimented with rational number replacements of  $n$  as an extension. (Do you know what happens for  $n = \frac{k}{2}$  for different choices of  $k$ ?)

The good news is that such experimenting and exploring leads to an attitude of conjecture making and testing. We want to encourage such powerful mathematical behaviors. It seems a natural outcome of using computer or calculator graphing utilities if you are experimentally inclined. The bad news is that ready access to graphing utilities does not necessarily lead to experimentation and exploration. In fact, teachers can be as didactically rule-oriented with exploratory tools as they were without. They can damp students' exploratory ventures. Teachers must instruct for the goal of conjecture making and testing; it doesn't happen automatically.

Students who matriculate in primary and secondary schools through mathematics dominated by the traditional three-step instructional process—giving a rule, working an example and directions to go do likewise with 30 similar exercises—need to be carefully introduced to exploration and experimentation as the mode for doing mathematics. Teachers, whether at the school or university level, need help in designing instructional activities to

promote the shift from rule-dominated mathematics to a more exploratory mode.

**The Information Base for Change.** The rigidity of many mathematics department faculty across the country in thinking about the use of technology in lower level mathematics courses is impressive. Harvey discusses the reasons for resistance to change identifying laziness, apprehensions concerning change, the fear of a mush-brain mentality being created within our students and the sheer magnitude of the problem of providing sufficient equipment in many of university settings.

It is hard to quarrel with Harvey's discussion of the reasons for mathematics programs failing to change. I would like to supplement and reinforce his arguments. Many mathematics faculty are not informed concerning effects of using technology in teaching. Research evidence is overwhelming in favor of using calculators. My colleague Marilyn Suydam who ran the National Institutes of Education supported Calculator Information Center for several years carefully collected information about effects of calculators on computational skills for several years. Her compilation of studies for which students were evaluated on paper-and-pencil computation but which compares performance of groups taught with calculator to those taught without summons attention. The results of more than 100 such calculator horse race studies are summarized in Figure 7. A betting person would favor the use of technology; 47 percent of the studies have the groups taught with the calculator winning on the paper-pencil evaluation tasks and only in seven percent of the studies does the paper-and-pencil group win the evaluation race.

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Calculators Lose	No Difference	Calculators Win
7%	46%	47%

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Figure 7. The Calculator Horse Race

Most faculty in mathematics departments do not read mathematics education research literature and are not familiar with such results. They argue for or against curricular and instructional

reform with no knowledge base often incorporating the specious logic of generalizing from a single personal instance. In an era of profound change in the environment for teaching and learning mathematics, mathematics faculty should read about the attempts to study systematically the effects of using technology in instruction in the literature of mathematics education. Dealing with the ignorance of colleagues while attempting to push through a proposal for a course modification consumes your creative energies that could better be invested elsewhere.

David Cohen made the interesting comment at the recent conference of the Psychology and Mathematical Education convention in DeKalb that we should not necessarily expect the research institutions to lead the way in innovation. Commitments to research and the correlated incentive and resource structures impose a barrier to change of instruction at such institutions. New equipment monies, for example, are invested to support research activities. More rapid implementation may be expected at those institutions more directly committed to investing resources in instruction than at the premiere research institutions until we arrive at a juncture where technological performance capability affects research productivity of faculty and students.

**Pressures for Change from Below.** A large number of schools and teachers have shown strong interest in the  $C^2PC$  precalculus course. Zalman Usiskin has noted comparable inquiry rate for School Mathematics Project materials that make extensive use of technology. Other evidence indicates that many teachers are making extensive use of technology in teaching. Experience suggests that the better high schools, the ones sending a major portion of students to tertiary level education, are the ones who have the resources of money and personnel to explore technology enhanced mathematics and are making pace setting ventures in curriculum and instruction. This is to say; a significant portion of the students to whom we would normally look as the source of good, sound mathematics majors enter the university with experience using technology in doing mathematics.

Query: How will such students behave in university mathematics courses that make no use of technology? Query: If you are required to teach technology-free mathematics, how will you treat such students? Query: Will such students elect to major in mathematics if they encounter freshmen and sophomore level courses that make no use

of technology particularly if other departments do feature technology in course work?

I think that if university mathematics faculties are not careful they will drive many excellent students to fields that have already joined the modern age. Students who come from school mathematics programs that use technology constitute a force for change at the college and university levels. Their expectations need to be honored or they will flee the Latinized curriculum.

An ancillary problem is specific to my field of teacher education in mathematics. Most teachers teach in the manner they were taught. Most of our prospective teachers will teach as they were taught. If our prospective teachers do not encounter use of technology in their mathematics classes, we cannot be surprised if they do not use technology in their teaching at the school level. Prospective teachers need regular, recurring encounters with technology in doing mathematics. We do not have enough time available in methods course work to make it happen without the constructive help of mathematics instructors. They need good models of use of technology in mathematics teaching to serve as a foundation for teaching.

**Representations and Generalizations in Mathematics.** I am speaking to the converted or as Harvey says, "the convicted." You know using technology changes how you do mathematics. For example, instead of expending hours generating the graphs of a very few parametric equations with pointwise plotting, in a small amount of time you examine several different parametric situations. Learning and using skills has been a labor intensive activity for our students. We have valued having those skills under sufficient control to allow students to operate efficiently; however, using them to extend ideas to new situations is remarkably difficult simply because of the nature of the skills.

Now, those hard won skills are not so important. We can examine many different but related examples easily. Often establishing a generalization has been difficult for a teacher because it takes so long to build the numerous exemplifications of an idea. Indeed, the finding of an instance may require a single computational process that is difficult to apply even though readily understood by a student. Generating the instance may have been so time consuming that the point is lost to the point of interfering with the cognitive processing required to form the generalization.

Our curriculum development efforts with C<sup>2</sup>PC and in the seventh and eighth grade level with the Approaching Algebra Numerically (AAN) project convince us that it is easy with the use of technology to build ability to generalize with students. They readily construct many instances. A numerical or graphical problem solving base is accessible to all students. They can extend ideas. In AAN, such a numerical problem solving base generated with the use of scientific calculators is used with good success to establish the idea of variable, a concept domain critical to understanding basic algebra. In C<sup>2</sup>PC, examination of many different instances allows students to fix the effects of changes in parameters in their thinking. Students discover and generalize ideas such as phase shift and amplitude change from graphs often before the instructor formally focuses their attention on the key idea. Transformations such as stretching, shrinking, translation and reflection are readily associated with shifts in parameters.

Teaching methodology changes as a result. Instructors find, according to our observations and the reports of teachers, that they focus on different aspects of mathematics. They are able to highlight making generalizations, problem solving and mathematical modeling.

Teachers report an ability to focus on other features of mathematics. We have given mathematics short shrift because of a narrow concentration on computational skills. Our C<sup>2</sup>PC teachers report that they are able to focus on communication in mathematics to a much greater extent. Learning to read mathematics has become more important in their classrooms. They think situational aspects of the technology generated graphs produces more talk about problems and representations resulting in sharpened ability to communicate mathematics. As teachers deal with the best ways to use the technology they place a premium on classroom and laboratory arrangements and management processes that extend the conversational, communicative aspects of dealing with the mathematics. We are pleased that teachers are making ventures and experiments to test different methodologies than they have used in previous, technology-free instruction.

Teachers find the ready capability to produce equivalent graphs of a problem situation with the modifications resulting from choices of viewing rectangles and other technology induced variants yields an interest and focus on representations. A natural premium arises in contrasting different algebraic and geometric representations of the same

functional relationships. This premium on representations builds mathematical modeling capabilities in the problem solving tool kit of students.

In summary relative to teaching, use of technology appears to force some changes in pedagogical orientation. A common pitfall to the novice in using technology is to apply blindly techniques of didactic rule-giving that are inconsistent with the experimental, exploratory possibilities inherent in the technology. Our experience indicates, however, that many instructors respond to the capabilities produced by technology to change teaching methodology and to value different, more powerful mathematical behaviors as outcomes of their instruction.

### Concluding Statement

We can Latinize mathematics at the university level or we can take advantage of technology. There is powerful impetus to change and, currently, strong pockets of opposition to what I trust is a natural evolution in curriculum and methodology. It is easy to forget that the examples we use and depend on in instruction via textbooks, blackboards and paper-and-pencil activities have evolved over generations. The mathematics community has a collective base of traditions in instruction of techniques and methods that serves to guide what we do. We are bound to make some mistakes in implementing technology due to having little such collective experience to guide problem selection and teaching methodology.

Curricular position statements such as *Toward a Lean and Lively Calculus* (1986) and *Calculus for a New Century* (1987) paint a picture of an undergraduate curriculum that does not fit present realities. Mathematics is more widely used and is applied in a variety of fields seldom represented in current instruction. Present realities include technological advances that should affect content selection and sequencing. This conference demonstrates the wide variety of ways in which the use of technology makes us rethink curriculum and instruction at the undergraduate level.

The NCTM's recommendations *Curriculum and Evaluation Standards for School Mathematics* (1989) prescribe a future for school mathematics consistent with the intents of *Toward a Lean and Lively Calculus* (1986) and *Calculus for a New Century* (1988). Themes of problem solving, communication in mathematics, and higher order cognitive functioning point toward products of the schools that should be quite different than those

students who are currently the targets of instruction at the undergraduate level. We must take advantage of the impetus for change in order to be ready for such students.

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