The Mathematical Physics of Rainbows and Glories

John Adam
Department of Mathematics and Statistics
Old Dominion University
Norfolk, VA 23529–0077
(To appear in Physics Reports)

Contents

Abstract

A detailed qualitative summary of the optical rainbow is provided at several complementary levels of description, including geometrical optics (ray theory), the Airy approximation, Mie scattering theory, the complex angular momentum (CAM) method, and catastrophe theory. The phenomenon known commonly as the glory is also discussed from both physical and mathematical points of view: backward glories, rainbow-glories and forward glories. While both rainbows and glories result from scattering of the incident radiation, the primary rainbow arises from scattering at about 138° from the forward direction, whereas the (backward) glory is associated with scattering very close to the backward direction. In fact, it is a more complex phenomenon physically than the rainbow, involving a variety of different effects (including surface waves) associated with the scattering droplet. Both sets of optical phenomena - rainbows and glories - have their counterparts in atomic, molecular and nuclear scattering, and these are addressed also. The conceptual foundations for understanding rainbows, glories and their associated features range from classical geometrical optics, through quantum mechanics (in particular scattering from a square well potential; the associated Regge poles and scattering amplitude functions) to diffraction catastrophes. Both the scalar and the electromagnetic scattering problems are reviewed, the latter providing details about the polarization of the rainbow that the scalar problem cannot address. The basis for the complex angular momentum (CAM) theory (used in both types of scattering problem) is a modification of the Watson transform, developed by Watson in the early part of this century in the study of radio wave diffraction around the earth. This modified Watson transform enables a valuable and accurate approximation to be made to the Mie partial-wave series, which while exact, converges very slowly at high frequencies. The theory and many applications of the CAM method were developed in a fundamental series of papers by Nussenzveig and co-workers (including an important interpretation based on the concept of tunneling) , but many other contributions have been made to the understanding these beautiful phenomena, including descriptions in terms of so-called diffraction catastrophes. The rainbow is a fine example of an observable event which may be described at many levels of mathematical sophistication using distinct mathematical approaches, and in so doing the connections between several seemingly unrelated areas within physics become evident.