# What's Math got to do with it?: 

Mathematics at the frontier of science and technology

## Technical Scholarships Weekend

 17 \& 18 February 2001 Old Dominion Universityadapted with permission from Professor Tony Chan Director, Institute for Pure and Applied Mathematics Department of Mathematics, UCLA

## Math Myths

- Math = terrifying


## Myth \#1: Math = Fear

## Los Angeles ©

ONTHEINTERNET: WWW.LATIMES.COM
CIRCULATION: 1,095,007 DAILY / 1.385,373 SUNDAY

## coummo one <br> Math <br> Equals Fear at 2-Year Colleges

- For many students, the
subject is a nemesis that blocks them from a career or transfer to a university.
Teachers struggle to provide help.

By JILL LEOVY
TIMES STAFF WRITER


## Some other Math Myths

- Math = terrifying
- Math $=$ static (Greeks, Newton, ...)
- Math $\neq$ other sciences
- Math = solitary
- Math = impractical as a career


## Math has an Image Problem

- Mathematicians - super-smart but "from another world"
- Actually, math lurks behind the curtain of popular subjects: simulation, forecasting, data mining, networks, polling, design, optimization, synthetic environments.
- Mathematicians have no monopoly on the practice of math - everyone (especially in science \& technology) does it.


## Intellectual Foci of the Sciences

| Field | The Study of ... |
| :--- | :--- |
| Mathematical Sciences | Patterns, structures, abstract models <br> of reality |
| Physics | Space-time, energy, matter |
| Chemistry | Molecular structure of matter |
| Biology | Genomics, organisms, ecologies |
| Materials Science | Materials, structures |
| Earth sciences | The earth: crust, core, oceans, <br> atmosphere |
| Astronomy | Origin and evolution of planets, <br> stars, and the universe |

## Common misconception



## Math at the Frontiers of Science



## Major Subfields of Math Sciences

| Subfield | The Study of ... |
| :--- | :--- |
| Foundations | Logical underpinnings of mathematics |
| Algebra and Combinatorics | Structures, discreteness |
| Number Theory and Algebraic Geometry | Properties of numbers and polynomials |
| Topology and Geometry | Spatial structures, patterns, shapes |
| Analysis | Extensions and generalizations of the <br> calculus |
| Probability | Randomness and indeterminate phenomena |
| Applied Mathematics | Modeling, analyzing, optimizing systems |
| Computational Mathematics | Experimental mathematics, computer based |
| Statistics | Analysis, application, and collection of data |

## Math in Society

| Problem/Application | Contribution from Mathematics |
| :--- | :--- |
| MRI and CAT Imaging | Integral geometry |
| Internet: search engines, compression | Graph theory, linear algebra, wavelets |
| Options valuation | Black-Scholes model and Monte Carlo simulation |
| Global reconnaissance | Signal processing, image processing, data mining |
| Confidentiality and integrity | Number theory, cryptology/combinatorics |
| Modeling of atmosphere and oceans | Wavelets, statistics, numerical analysis |
| Analysis of the human genome | Data mining, pattern recognition, algorithms |
| Rational drug design | Data mining, combinatorics, statistics |
| String Theory (Theory of Everything?) | Geometry |
| Aerodynamic design | Differential equations, optimization |
| Earthquake analysis and prediction | Statistics, dynamical systems/turbulence |

## An Ancient Subject



Pythagorus of Samos
569-475 B.C. (?)
musician, geometer,
first "pure" mathematician

## A Modern Subject



Karen Uhlenbeck, 1942 -
(University of Texas,
National Academy of Sciences)
expert in partial
differential equations

## Why is Math Ubiquitous?

- Math allows description, analysis, and prediction (simulation) of quantitative systems
- Math exposes structures \& patterns of nature
- Math leverages wisdom, through abstraction
- Most physical laws are expressed in mathematics: Newton's laws, Maxwell's equations, Schrodinger's equation, Einstein's relativity, etc.
- Norbert Wiener: "The unreasonable effectiveness of math..."


## Advances in Computing not just Hardware

- For the solution of important differential equations "... the progress made through better methods from 1945 to 1978 exceeds the progress made through faster computers " by 40 times*
- Factoring a large integer using modern mathematical techniques would be 120,000 times faster than using techniques from the 1970's. So if it took 1 day to do the problem now, it would have taken over


## Progress


$\square$ Better Hardware Better Algorithm

* From "Numerical Methods, Software, and Analysis", 1983, by John R. Rice, Purdue University.
**From "The future of integer factorization", 1995, by Andrew Odlyzko, AT\&T Bell Laboratories.


## An Example of a Math Proof

Theorem: There is no largest prime number.
Proof: Assume there is a largest prime number. List all the primes as $\mathrm{p} 1, \mathrm{p} 2, \ldots ., \mathrm{pn}$. Let $\mathrm{p}=\mathrm{p} 1$ * $\mathrm{p}^{*}$.... $\mathrm{pn}+1$.

Then p is prime because no prime divides into it. Also, p > pn. Hence, contradiction!
E.g.: Suppose 7 is largest prime. Then $2 * 3 * 5 * 7+1=211$ is also prime.

## An Example of "Pure" Math: Fermat's Last Theorm

The equation:

$$
X^{n}+Y^{n}=Z^{n}
$$

has no integer solutions for $\mathrm{n}>2$.
(Note: $\mathrm{n}=2$ has many solutions - Pythagoras)

## What's playing in classrooms and laboratories near you?

- Functional Genomics
- Finance
- Communication Networks
- Modeling and Simulation of physical systems
- Image processing


## Functional Genomics

The Protein Folding Problem

A hidden Markov model for sequence analysis


Slides from "Molecular biology databases", by Terry Speed, based on Chapter 2 of "Post-genome Informatics" by Minoru Kanehisa, Oxford University Press, 2000.

## Financial Mathematics

- Nova documentary about the Black-Merton-Scholes Formula. The film tells the fascinating story of the invention of the Black-Scholes Formula, a mathematical "Holy Grail" that forever altered the world of finance and earned its creators the 1997 Nobel Prize in Economics.


The Formula That Shook the World

$$
\text { hot science } \sim \text { Play a Virtual Malket }
$$ Impact of Online Traifing

A Trader's Lexicon


## Communication Networks

MIT Professor of Applied Mathematics Tom Leighton, who had an office down the hall from web guru Mr. Berners-Lee, was intrigued by a challenge from the latter. ... Leighton recognized that a solution to web congestion could be found in applied mathematics and algorithms.
Leighton and graduate student Lewin were joined by other scientists with expertise in computer science and data networking to develop the mathematical algorithms necessary to handle the dynamic routing of content - now a $\$ \mathbf{5}$ billion NASDAQ company.


## CUSTOMERS:

Abercrombie \& Fitch
Adobe
Apple
C-SPAN
CBS
Chicago Sun-Times
CNN Interactive
GO Network
iVillage.com
KBkids.com
LA Times
Lands' End
Lycos
Monster.com
Oxygen Media
Reuters
Times Company
Digital
Washington Post
Yahoo

## Math Modeling \& Simulation of Crystal Growth



STM micrograph

atomistic model

-Goal: Improved design and fabrication of semiconductor materials
-Strategy: Mathematical modeling, numerical simulation, feedback control
-Applications: satellite communications, CD players, cell phones, lasers.

## Image Processing: Inpaintings

Scratch Removal


Disocclusion


Graffiti Removal


## Synthetic Images



## What's Math got to do with it



Picture courtesy of Doug Roble, Digital Domains.

## TECHNOLOGY

## SCIENTIFIC

Physical Phenomena

Experiments


Inexpensive

Just do it!


## ENTERTAINMENT

Physical or Artificial Phenomena


Effects Shots


Or
Impossible
Inexpensive

Just do it!

## Simulations

## Newt Gingrich Letter (9/28/2000)

As the former NIH director, Harold Varmus indicated, the biological research is dependent on continuing breakthroughs in basic science-math research. Over the last 7 years (from fiscal 1994 through 2001) we have increased the NIH research budget from $\$ 11.544$ billion to $\$ 19.729$ billion. That is an increase of $71 \%$ since 1994. By contrast we have increased the NSF research budget in the same period only from $\$ 2.472$ billion to $\$ 3.134$ billion or an increase of $27 \%$. This will ultimately lead to an atrophying of our investment in math, physics, chemistry and other basic knowledge and then to a decline in our national security, in our economic growth, and in our ability to do medical research. Our current economy is a reflection of past investments in scientific research (the computer chip and the internet are only two examples of government funded progress).

## Harold Varmus Letter (10/4/2000)

Scientists can wage an effective war on disease only if we--as a nation and as a scientific community-harness the energies of many disciplines, not just biology and medicine. The allies must include mathematicians, physicists, engineers and computer and behavioral scientists. I made this case repeatedly during my tenure as director of NIH, and the NIH has made significant efforts to boost its support of these areas. But in the long run, it is essential to provide adequate budgets for the agencies that traditionally fund such work and train its practitioners. Moreover, this will encourage the interagency collaboration that fuels interdisciplinary science. Only in this way will medical research be optimally poised to continue its dazzling progress.

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The writer is president of Memorial Sloan-Kettering Cancer Center and a
former director of the National Institutes of Health.
He received the Nobel Prize in Medicine in 1989.
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## Mathematics \& Statistics at ODU

- One of about 25-30 primarily Applied and Computational Departments in the US
- 28 faculty for about 40 majors and 30 graduate students (and thousands of student clients from other departments!)
- 3 UG majors: Mathematics, Statistics, Mathematics Education
- Core department in new Computational Sciences \& Engineering Initiative


## Sample Research Projects in Math \& Stat at ODU

- Human radiation risk abatement in spacecraft
- Parallel computer algorithms for optically thick radiation transport
- Dynamics of protein folding (optimal drug design)
- Prediction of noise from ducted fan-jet engines
- Digital image compression
- Survival analysis of heart transplants
- Prostate cancer bio-markers


## Graduate School and Career Opportunities for Math Majors



## Do New Math — Win a Million Bucks!

- The Clay Mathematics Institute has just created the Millenium Prize; see http://www.claymath.org/prize_problems
- Seven problems that have defied solution for ages are singled out for $\$ 1$ million in prize money each (Poincaré conjecture, Riemann hypothesis, etc.)
- The proof of Fermat's last theorem (from the 1650's) in 1993 has reinvigorated such quests


## Math Myths Revisited

- Math = fear
- Math $=$ static $($ Greeks, Newton, ...)
- Math $\neq$ other sciences
- Math = solitary
- Math = impractical as a career


## Math Myths Debunked

- Math = joyous
- Math = dynamic (internet, genomics, ...)
- Math $=$ at the frontiers of other sciences
- Math = collaborative
- Math = foundational for evolving careers
- Math = "cool"


## Mathematics is Empowering!

See you in class in the Fall!
http://www.math.odu.edu


