THE OLD DOMINION UNIVERSITY CALCULUS PROJECT

Przemyslaw Bogacki, Gordon Melrose and Philip R. Wohl
Department of Mathematics and Statistics
Old Dominion University, Norfolk, VA 23529
E-mail: bogacki @ math.odu.edu, melrose @ math.odu.edu

1. Introduction.

Is there a need for another calculus project? At the outset of this project, in the fall of 1991, we thought the answer was no. We intended to submit a proposal to the state council for higher education in Virginia (SCHEV) for funds to initiate technology-based reform in the Old Dominion University calculus sequence. We fully expected to adopt one of the existing projects, either in toto or with only minor modifications. This, however, was not the case.

The proposal to SCHEV was funded, and we received $135,750 over a two year period. This was supplemented by $100,460 in matching funds by ODU. By attending workshops on the Duke and Purdue projects, and attending numerous conferences, we realized the need for evolutionary change in the calculus sequence. In our project, we have tried to supplement the existing calculus sequence with a computer lab component to emphasize graphical and numerical aspects of calculus. Furthermore, we try to emphasize fundamental concepts with the lab material, at the expense of "real-life" or "challenging" problems.

Section 2 outlines our reasons for not adopting one of the existing projects. In section 2, we give our reasons for choosing Mathcad for Windows (supplemented with Maple V) and for choosing workstations configured in a local area network. In section 4, we describe our revised calculus curriculum and the advantages of teaching in an "electronic classroom". At the core of our project are computer-based laboratory asssignments which emphasize a numerical and graphical understanding of calculus. Excerpts from several laboratory assignments are presented in section 5. In conclusion, some preliminary assesment data is reported in section 6.

2. Should we adopt an existing calculus project?

During the spring and summer of 1992, we attended workshops and gathered information on the Purdue, Duke and Harvard projects. We also reviewed the formats and laboratory materials of several projects developed at other universities including RPI, the University of New Mexico, and the University of Illinois. Adopting one of these established projects would clearly have provided several advantages:

- **Instructional Material Already Developed And Field Tested.**

The existing projects had developed a wealth of instructional material over several years; therefore, one might expect that with minor modifications an existing project could be successfully implemented at ODU.

- **Funded Workshops Offered By The Major Projects.**

Each of the major projects had workshops for individuals interested in adopting their calculus project. This would be an effective method to become familiar with the philosophy and content of a project in a short period of time.

- **Network Of Peers To Aid In Troubleshooting And Problem Solving.**

Finally, and perhaps most importantly, existing projects would provide a forum for faculty to communicate about problems and solutions encountered in their implementation. This support feature would help to avoid repeating mistakes made at other institutions. The methods employed in disseminating information included reunions of workshop participants, conferences, electronic networks and newsletters.

Notwithstanding these advantages, it was apparent that critical problems would arise if we adopted any one of these projects "as is" at Old Dominion University. The vast majority of students enrolled in Calculus I at ODU are required (by a placement test administered to all freshmen) to take one or perhaps two semesters of precalculus. While this is typical at many institutions, it is not the case at schools where the major projects were developed. Therefore, functions and other precalculus topics constitute a substantial portion of Calculus I in many of those projects. For us this would mean either duplicating a large portion of our precalculus sequence, or revising both the precalculus and calculus curriculum.

The first approach would obviously cause topics (e.g. techniques of integration) currently covered in Calculus I to be delayed to Calculus II. Delaying such topics would be incompatible with existing co-requisites in engineering and science classes. To fully implement such an approach would require prior approval of two colleges and many departments. We did not see this as a tractable solution.

There is a need for technology-based reform in the precalculus sequence, and there are many such undertakings currently underway. Much of the material in the major calculus reform projects is worthy of inclusion in this effort. However, at ODU and many other institutions, the concepts of function and related topics belong in a separate precalculus course. Moreover, we felt that simultaneously undertaking both a precalculus and calculus reform project was not feasible.

Another perceived problem was that several of the major calculus projects emphasize the use of technology in solving "applications problems" to motivate students in their study of calculus. Although originally embracing this approach, we shifted to a position of using the computer to emphasize fundamental concepts in calculus. We felt that the applications approach might have worked well for us if the majority of our students were not weak in precalculus skills. However, even after a traditional precalculus course many are still weak in algebra. Moreover, the conceptual content of calculus already presents a sufficient challenge to most of our students. For the same reason, we felt that the early introduction of such topics as O.D.E.'s and predator-prey models would not excite or motivate the average student. Rather, it would merely put another obstacle between him/her and successful completion of the course.

At some large universities the calculus curriculum varies markedly across different sections of the same course. Similarly, at many small colleges these options are often left to one or two individuals who regularly teach calculus. At such institutions even the most experimental approaches can be tried on a pilot basis with little chance of risking complete failure.

At Old Dominion University, however, 20-25 sections of Calculus I and II are taught each year, there is a uniform syllabus and common final examination, and the fifteen or more faculty who regularly teach these courses must agree on any major curriculum changes. In any department where such circumstances hold, revolutionary reform projects stand little chance of winning sufficient support of the faculty. This makes it either impossible or too painful for such departments to adopt those projects. For this reason, we opted for a more evolutionary approach by integrating computer technology into the lecture format (while maintaining the traditional curriculum and text) and by creating computer-based laboratory assignments which emphasize a numerical and graphical understanding of calculus.

3. Which computing environment should we use?

Early in the decision process, we opted for IBM-PC compatible computers. Compatibility with existing hardware, budget limitations and the availability of software were the deciding factors. On this hardware platform, the mathematical systems we considered were Derive, Maple V release 2, Mathematica and Mathcad 3.1. When comparing these systems, the most important criterion was the extent to which they would help us reach two goals:

- **Enhance Students' Graphical And Numerical Understanding Of Calculus.**

- **Make The Software/Hardware Aspect Of The Course As Transparent To A Student As Possible.**

In our view, while all of these systems can meet the first goal, Mathcad far exceeds the others in meeting the second goal.

Mathematica and Maple are sophisticated systems which require a substantial investment in time for a user to learn their complex features. This means that during the first semester an average student will struggle to learn the system's intricacies, while also struggling with calculus. In contrast, Derive has a menu-driven interface which puts most of its functionality at the user's fingertips. Although it doesn't take a long time to acquire a working knowledge of Derive, it also doesn't take a long time for a student to outgrow it. It has been our experience that Mathcad is positioned "about right" with respect to its ease of learning and functionality, at least as far as univariate calculus is concerned.

Mathcad has important advantages over its competitors in the areas of document structure and user interface. While students may learn by just performing activities on screen without saving their work, we feel that learning is significantly enhanced by placing an emphasis on the composition of laboratory reports.

A document in Maple, Mathematica or Derive has a one-dimensional structure, which consists of a sequence of input commands and output. Derive, unlike the other systems, allows insertion of an object (whether input or output) only at the end of the document. In contrast, a Mathcad document has the two-dimensional structure of a spreadsheet with a fine, invisible grid. Therefore, unlike Maple, Mathematica or Derive, two or more objects in Mathcad can coexist next to one another. The order of processing commands within the document takes place from top line to bottom line, and left to right within each line. In creating laboratory assignments and classroom demonstrations, this two-dimensionality allows us to present ideas, patterns and relations by grouping together various tables, graphs and formulas. This "freedom of form" challenges and motivates students to think about the best way to arrange information in their documents. Additionally, when grading student lab assignments in electronic form we can type several lines of comments, or brief remarks in the margin, without interrupting the flow of the document.

Mathcad does have several shortcomings. Most importantly, from the viewpoint of teaching Calculus II, Mathcad lacks both implicit and multiple plots in 3-D graphing, and animation facilities. Therefore, for some portions of Calculus II we are using Maple's superior graphing capabilities. Also, although Mathcad 3.1 supports some symbolic features (a subset of Maple), these features are not totally integrated into the system. For example, expressions to be processed symbolically must be stated explicitly. A user-defined function will always be treated as an unknown function by the symbolic processor.

Two windowing features are essential for our purposes. As stated earlier, a number of Calculus II assignments make use of Maple V to produce 3-D graphics and animations. Therefore, windowing allows us to keep our Mathcad document open while working in Maple. In addition, the clipboard allows us to copy and paste Maple plots into our Mathcad document.

The workstations in our laboratory consist of 486/DX computers configured in a local area network, using Novell NetWare version 3.11 software. This arrangement is clearly advantageous with respect to system maintainability and security. The network also makes it possible for students to store their documents, submit their reports for grading, and receive graded documents electronically without having to use paper or floppy disks. Moreover, students are easily provided with electronic handouts in the form of Mathcad documents (answer keys to class tests, demonstrations, messages).

4. Computer-based Calculus at Old Dominion University.

Having answered the two key questions addressed in Sections 2 and 3, we then focused our deliberation on the many decisions to be made in preparation for teaching three pilot sections of Computer-based Calculus I in the spring of 1993. These decisions were related to the lecture/lab format, content of the syllabus, creation of laboratory assignments, testing, grading, student lab assistants, project assessment, and how to win the support and participation of other faculty (arguably the most critical aspect of any reform project). The results of our deliberations are summarized in this section. Further discussion of our laboratory assignments and project assessment is contained in Sections 5 and 6.

Computer-based Calculus I and II at Old Dominion University is taught in "electronic classrooms" wherein both computer-integrated lectures are given by the instructor, and the computer lab assignments are completed by the students. In addition to being an efficient use of our limited instructional space on campus, this arrangement allows the instructor to give computer demonstrations while also having the students gain "hands on" experience during any class meeting. Moreover, we have found that regular use of the computer in class is important for students to clearly see how the lab assignments are related to the lectures.

Calculus I (through techniques of integration) and Calculus II (through multiple integrals) are each five credits, with four hours per week devoted to lectures interspersed with computer demonstrations. Each instructor has the electronic classroom reserved as a "closed lab" an additional two hours per week in order to start his/her class (consisting of 30 to 40 students) on the lab assignment for that week. Students are paired into teams and work together on a computer. Teammates are encouraged to assist one another in understanding and completing each lab assignment, and receive a team lab grade. Lab assignments average three hours in length and are completed during "open lab" hours. An instructor and assistant (or two assistants) are assigned to the lab whenever the lab is open. As a result of participation in lab activities, each student is expected to have acquired some proficiency in using Mathcad (and

Maple in Calculus II). At about midsemester, every student is required to *individually* pass a test of basic Mathcad and/or Maple skills. The final course grade is determined as follows: lab assignments-25%, class tests-40%, and uniform final examination-35%. The class tests and final exam are "closed book, no computers." However, some of the questions are related to

Laboratory 6

Spring 93
MATH 207

Riemann Sums and the Fundamental Theorem of Calculus

Objective

To emphasize that a definite integral is a number which is defined to be the limit of a Riemann sum. In this Lab you will use Mathcad to evaluate definite integrals using left and right Riemann sums, the Midpoint Rule and the Trapezoidal Rule. You will also verify the Fundamental Theorem of Calculus.

Prerequisites

Read Sections 5.1 through 5.4 and 5.6 of the textbook.

Figures 1. and 2. below show rectangles associated with **the left and right Riemann sums** of the function f(x) = x^2 . In both figures, the interval a=0 to b=1 is partitioned into n=5 subintervals of width $\Delta x = 0.2$.

$f(x) := x^2$ $\quad n := 5$ $\quad a := 0$ $\quad b := 1$ $\quad \Delta x := \frac{b-a}{n}$ $\quad x := a, a + 0.03 .. b$

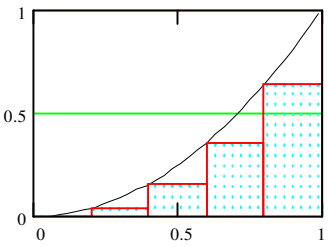

Figure 1.

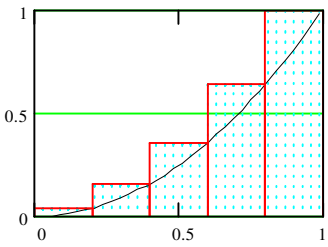

Figure 2.

You can use Mathcad to calculate the left Riemann sum associated with Fig.1 by first setting the range variable

$$xleft := a, a + \Delta x .. b - \Delta x$$

to cover all the x values where f should be evaluated, and then computing the sum:

$$LeftSum = \sum_{xleft} f(xleft) \cdot \Delta x = 0.24$$

EXERCISE 1 Calculate the right Riemann sum, RightSum . (First define the xright range variable.)

Hint: How is xright related to xleft ?

Figure 1. The beginning of the laboratory assignment on Riemann Sums and the Fundamental Theorem of Calculus.

topics developed in lab assignments. Lab assignments are not scheduled during the same week as class tests. Therefore, we have time for about eight lab assignments per semester.

As mentioned in Section 2, our lab assignments emphasize a numerical and graphical understanding of calculus. They are either given in electronic form, wherein the student is required to "fill-in-the-blanks," or in hardcopy so that the student is required to create his/her own Mathcad document to answer the questions. To date, we've created and class-tested the following labs: Mathcad Basics, Limits, The Derivative, The Chain Rule and Implicit Differentiation, Curve Sketching, Riemann Sums and the Fundamental Theorem of Calculus, Logarithmic and Exponential Functions, Introduction to Symbolics, Optimization, Sequences, Series, Taylor Polynomials, Parametric Equations in the Plane, Polar Curves, Lines and Planes, and Vector-Valued Functions. A portion of an electronic lab is presented in Section 5. Two examples of how we use Maple's graphing facility, as they appear in the labs, are also shown in Section 5.

Three sections of Computer-based Calculus I and two sections of Computer-based Calculus II are being taught each semester during 1993-94. To win the support and participation of other faculty, a week-long workshop was held in May, 1993 to discuss the goals and philosophy of the new calculus sequence. During this workshop, faculty worked through lab assignments and provided valuable feedback. A follow-up one day workshop will be held in December, 1993 to review the continuing evolution of the project.

EXERCISE 2 (a) Complete the table below. In the first row, type the values of the left and right Riemann sums obtained above for n=5. Then recalculate the sums for n=10,20,50 and enter their values in the table.

f(x)= x^2 on [0,1]

n	left Riemann sum	right Riemann sum
5		
10		
20		
50		

Figure 2. Part (a) of Exercise 2.

$f(x) := x^2$ $n := 10$ $a := 0$ $b := 1$ $\Delta x := \frac{b-a}{n}$ $x := a, a + 0.03.. b$

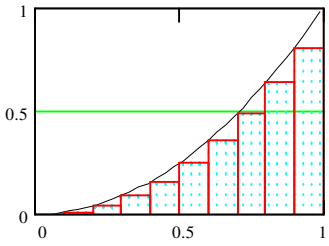

Figure 1.

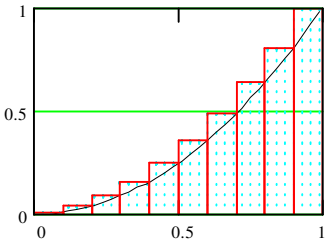

Figure 2.

You can use Mathcad to calculate the left Riemann sum associated with Fig.1 by first setting the range variable

$$xleft := a, a + \Delta x.. b - \Delta x$$

to cover all the x values where f should be evaluated, and then computing the sum:

$$LeftSum = \sum_{xleft} f(xleft) \cdot \Delta x = 0.285$$

Figure 3. A portion of the lab document on Riemann sums (included in Fig. 1) after changing the value of n to 10.

5. *Examples from the Laboratory Assignments*

This section includes excerpts from several laboratory assignments we used in computer-based Calculus I and II in the spring and fall semesters of 1993.

Our first example is a portion of an electronic laboratory assignment on Riemann Sums and the Fundamental Theorem of Calculus.

n	left Riemann sum	right Riemann sum
5	0.24	0.44
10	0.285	0.385
20	0.309	0.359
50	0.323	0.343

Figure 4. Completed table in part (a) of Exercise 2.

(b) Look at the table you have just completed. As n increases, what happens to the left Riemann sum values? Does the left Riemann sum appear to be converging? What about the right Riemann sum values? What can be said about the value of the actual area bounded by f(x) and the x-axis between x=a and x=b?

How are these findings related to Theorem 5.5 on page 276 of your text? In particular, how are the left or right sums related to the upper or lower sums discussed there?

(c) By hand calculation, find an antiderivative of f(x) = x^2 and use Mathcad to define it to be the function F(x).

Refer to the definition of the definite integral (p. 284) and to the Fundamental Theorem of Calculus (Theorem 5.11, p. 291) . What does the theory say about the relationship between the limits you obtained in (b) and the function F(x)? Use Mathcad to numerically verify that this relationship holds.

Figure 5. Part (c) of Exercise 2 of the laboratory on Riemann sums.

Consider the motion of two spinning ferris wheels described by the parametric equations

$$x1(t) = 20\cdot\sin\left(\frac{\pi\cdot t}{20}\right) \qquad y1(t) = 20 - 20\cdot\cos\left(\frac{\pi\cdot t}{20}\right)$$

$$x2(t) = 45 + 15\cdot\sin\left(\frac{\pi\cdot t}{15}\right) \qquad y2(t) = 15 - 15\cdot\cos\left(\frac{\pi\cdot t}{15}\right)$$

(a) Use Mathcad to obtain the graphs of both ferris wheels on the same plot. Of course the graphs are really circles. You should stretch the plot horizontally so they appear circular.

(b) Use "Alt-Tab" to switch to Program Manager, and invoke Maple. Load the file `LAB04C.MS` from the `G:` drive and execute it.

Play and view the animation which shows the location of two points (x1(t),y1(t)) and (x2(t),y2(t)) on the spinning ferris wheels starting at t=0. Play it again, stopping the animation when the two points appear closest. Locate the coordinates of these two points by placing the tip of the pointer on them and clicking with the left mouse button. *Write down their values for use in part (c) of this problem.* Copy and paste the plot into your Mathcad document. Exit from Maple .

(c) Use Mathcad to calculate the distance between the two points obtained in part (b).

(d) The value of the distance obtained in part (c) is only a rough estimate of how close the two points come. Use Mathcad to obtain a more accurate estimate of the minimum distance between the two points.

Figure 6. An excerpt from a laboratory assignment on Parametric Curves, where Maple animation facility is used.

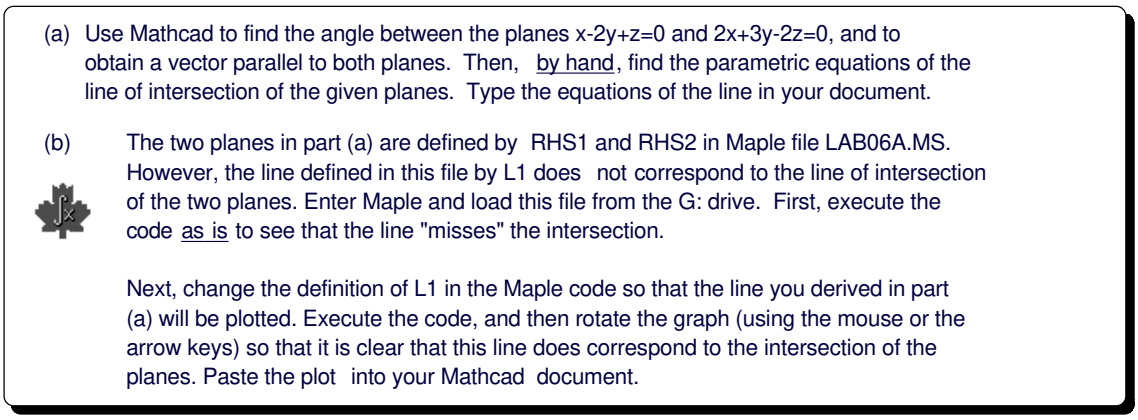

Figure 7. An excerpt from a laboratory assignment on Lines and Planes, where Maple graphing facility is used.

Figure 1 contains the opening portion of the assignment (note that certain parts of the document have been omitted from the figures to improve clarity). Having read the information contained in Figure 1 and having typed in the answer to Exercise 1, students then turn to Exercise 2 contained in Figure 2.

After typing in the values of the left and right sums corresponding to n=5, the students are expected to go back to Figure 1 and modify the value of n. Figure 3 shows what they see for n=10. Notice that not only are the graphs affected by the change in the value of n, but so is the value of the left Riemann sum expression (and right Riemann sum in Exercise 1).

For each modification of n, students view how the plots change, and then record the values of the left and the right Riemann sums in the table. Figure 4 shows the completed table of part (a) of Exercise 2. Parts (b) and (c) of the same exercise (shown in Figure 5) lead to the Fundamental Theorem of Calculus.

We close this section with two examples, presented in Figures 6 and 7, which illustrate how we use Maple's graphing facility in the laboratory assignments.

6. Assessment.

With regard to project assessment, we decided that four measures of evaluation would be useful: (1) questionnaires to determine student attitudes, (2) performance on tests and lab exercises to determine conceptual understanding, (3) comparison of scores on a common final given to computer-based and traditional sections of calculus, and (4) long-range impact (grades in subsequent courses, senior exit exams and interviews).

As mentioned earlier, the first three sections of computer-based Calculus I were taught at Old Dominion University in the spring of 1993. In the fall of 1993, we are teaching two sections of computer-based Calculus II, in addition to three computer-based Calculus I sections. As such, it is far too early to have a complete and detailed assessment of the project. Some preliminary data, however, was obtained at the end of the spring semester from student responses to a questionnaire given out to the three sections of computer-based Calculus I.

Of the 54 students who took the final, 45 completed the questionnaire. Responses from a portion of the questionnaire were as follows:

A *Strongly Agree;* B *Mildly Agree;* C *Mildly Disagree;* D *Strongly Disagree;* E *No Opinion*.

	A	B	C	D	E
I enjoyed the course	44%	51%	2%	2%	0%
The computer lab assignments were interesting	51%	40%	7%	2%	0%
I was able to learn basic features of Mathcad in a short time	87%	13%	0%	0%	0%
I found Mathcad easy to learn	76%	22%	2%	0%	0%
The use of Mathcad in this course has improved my problem solving skills	33%	49%	9%	9%	0%
The use of Mathcad revealed aspects of calculus that I had not thought about before	42%	40%	0%	7%	11%
Using a computer made learning calculus more interesting	69%	22%	7%	2%	0%
The computer labs appear to have a positive impact on my grade	73%	16%	4%	2%	4%
The computer labs had a positive impact on my overall understanding of calculus	60%	29%	4%	7%	0%

Therefore, it appeared from the responses that:

- **Students Found The Computer-Based Calculus Interesting And Enjoyable.**
- **The Windows Version Of Mathcad Was Easy To Use.**
- **The Use Of Mathcad Improved The Learning Of Calculus.**
- **Computer Labs Improve Students' Perception Of Their Calculus Skills.**