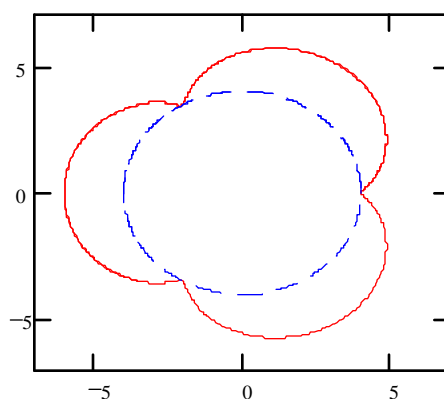

10 Plane Curves, Polar Curves

In this chapter the emphasis is on curves in the plane which are not represented by an explicit functional relation of the form $y = f(x)$. An important method of describing such curves is parametric equations, where each of the spatial variables (x, y) is given in terms of another variable, the parameter (t) . For example consider the curve traced out by a point P on the circumference of a circle rolling around the outside of another circle. Such a curve is called an epicycloid and is illustrated below.



Trying to describe one variable in terms of the other in this case leads nowhere. However, if we try to describe the position of the point P in terms of angles (parameters) the point P makes with the centers of the circles, we can arrive at a set of parametric equations for the motion.

A special case of parametric equations arises when we use polar coordinates and describe curves in terms of polar variables r and θ .

This chapter includes two laboratory assignments. The first assignment emphasizes parametric equations in general. Using Mathcad will enable you to graph and find arc lengths and areas enclosed by curves given parametrically. The second laboratory assignment is devoted to polar curves and again focuses on arc lengths and areas enclosed by polar curves.

10.1 Activity: Parametric Curves in the Plane

Prerequisites: Read Sections 10.1 - 10.2 LHE.

In this activity, you will use Mathcad and Maple to visualize and enhance your understanding of the behavior of some interesting parametric curves in the plane.

Instructions

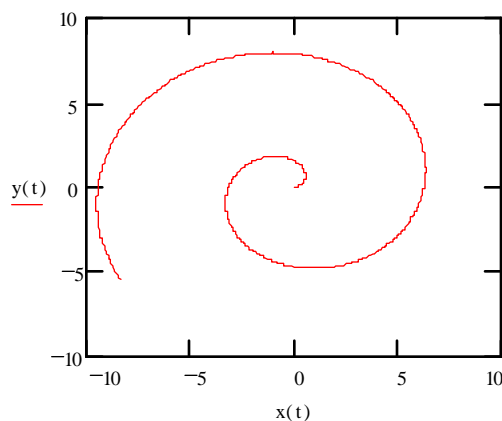
After reading the comments and studying the worked examples, open a blank Mathcad document

and create your report there. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

- Throughout this chapter, you will use Mathcad to plot parametric curves in the plane - here's how this is done. First, define the functions $x(t)$ and $y(t)$, as well as the parameter t range variable. Then, select **Create X-Y Plot** from the **Graphics** menu and fill the middle x -axis placeholder with $x(t)$ and the middle y -axis placeholder with $y(t)$. After you hit the <Enter> key, the graph of your parametric curve should be obtained, as in the following example

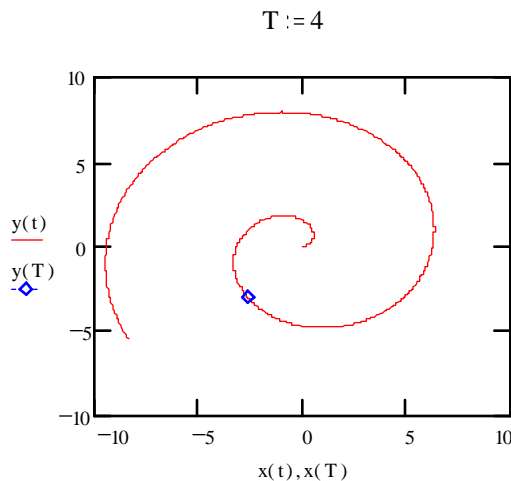
$$x(t) := t \cdot \cos(t) \quad y(t) := t \cdot \sin(t) \quad t := 0, 0.01.. 10$$



Note that we are using the same Mathcad graphing facility as that used in plotting functions $y = f(x)$. Consequently, all the features of Mathcad plots that we discussed earlier, such as the graph format, zoom and crosshair, can be used in conjunction with parametric plots.

- A parametric plot provides a convenient way to visualize the relationship between the functions $x(t)$ and $y(t)$. It does not, however, reveal the correspondence between the values of the parameter t , and the point $(x(t), y(t))$.

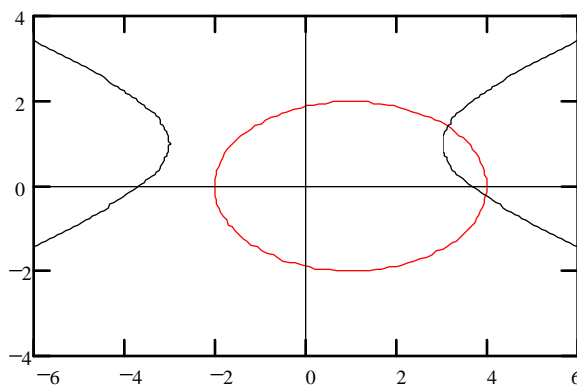
In Mathcad, we can use add the point to the graph to illustrate that correspondence:



We first define the value of T which we are interested in (this variable name is capitalized so that it won't interfere with our range variable t). Then, add $y(T)$ after $y(t)$ and $x(T)$ after $x(t)$ (separating the two by a comma in both cases). Finally, double-click on the graph, and change the "Symbol" for the second trace to a diamond ("dmnd").


By changing the value of T above the graph, you will now be able to "trace" the graph for varying parameter values.

3. Numerous times in this manual, we invoke Mathcad's "root" function to find a numerical approximation to the solution of an equation. In this activity, we will be asking Mathcad to solve *systems* of equations. While this cannot be accomplished using the root function, Mathcad has the "Given ... Find" *solve block* which can be used for that purpose. The following example illustrates this approach. We determine the intersections of the ellipse $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{(y-1)^2}{2} = 1$



$x := 3.5$ $y := 2$ <----- Define initial values of x and y (must be done before the keyword "Given" appears)

Given

$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$ <----- List all the equations after the keyword "Given" (use the button  to set up an equation)

$\frac{x^2}{9} - \frac{(y-1)^2}{2} = 1$

$\text{Find}(x,y) = \begin{pmatrix} 3.125 \\ 1.412 \end{pmatrix}$ <----- Typing Find(x,y) followed by the "=" key asks Mathcad produce the solution. Values are reported in the same order in which the variables were listed after the "Find" keyword.

Notice that the initial values of the unknowns should be close to the solution being sought. Consider the following example:

$x := 4$ $y := 0$ <----- Different initial values result in ...

Given

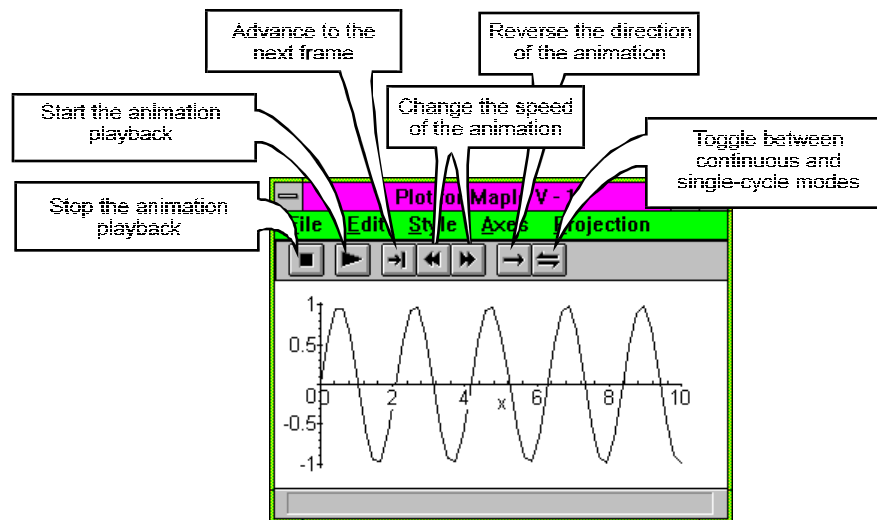
$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$

$\frac{x^2}{9} - \frac{(y-1)^2}{2} = 1$

$\text{Find}(x,y) = \begin{pmatrix} 3.98 \\ -0.233 \end{pmatrix}$ <----- ... a different answer (this corresponds to another intersection point).

If the initial guess you provided was not sufficiently close to a solution, then Mathcad may report an error. (Recall that similar consideration has to be given to an initial guess when using Mathcad's root function.)

- In this activity, you will be asked to view animations generated using Maple. For reference, here is the layout of Maple's animation window with a definition of each of its panel buttons.



You can have these definitions displayed in Maple's status line (at the bottom) if you click and hold the left mouse button on the appropriate area of the panel.

Example 1

If a circle of radius a rolls along a straight line, then the curve traced by any given point on the circumference of the circle is called a cycloid. If the given point is initially at the origin, then the parametric equations which describe the cycloid (derived on p.672 LHE) are given by

$$x = a(t - \sin t) \quad \text{and} \quad y = a(1 - \cos t).$$

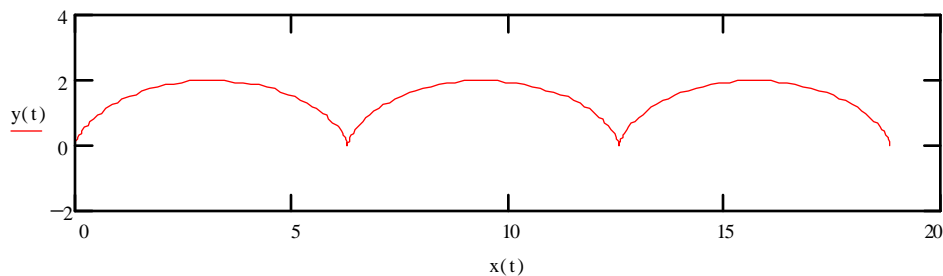
Plot the cycloid for three revolutions of a circle with radius 1. Based on the geometric interpretation of the cycloid in terms of a rolling circle, determine the location of the maximum points and cusps on the graph. Replot the cycloid with $a = 3$. Has the shape of the cycloid changed? What about its size?

Solution

$$t := 0, 0.1..6\pi \quad a := 1 \quad x(t) := a \cdot (t - \sin(t)) \quad y(t) := a \cdot (1 - \cos(t))$$

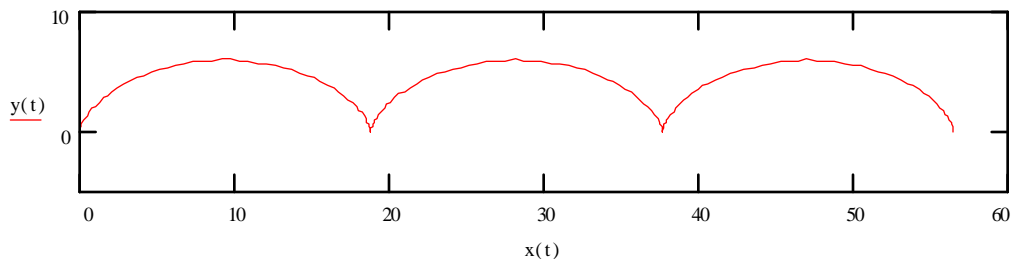
Maximum points are at $(\pi, 2)$, $(3\pi, 2)$ and $(5\pi, 2)$. (To find, set $dy/dt = 0$.)

Cusps are at $x = 0, 2\pi, 4\pi$ and 6π . (To find, set $y=0$.)



$$a := 3 \quad x(t) := a \cdot (t - \sin(t)) \quad y(t) := a \cdot (1 - \cos(t))$$

The value of the radius, a , only affects the size of the graph, not its shape, because it only appears as a multiplier in the expressions for x and y .

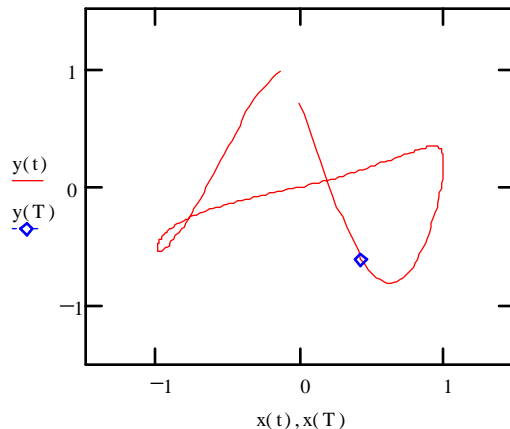


Example 2

- (a) In Mathcad, graph the parametric curve with the equations $x(t) = \sin 3t$ and $y(t) = t \sin(4.5t^2 + t + 1)$ for $-1 \leq t \leq 1.05$. Follow the procedure described in Comment 2 to show the point on the curve corresponding to $T = 0.9$.

Solution

$$x(t) := \sin(3 \cdot t) \quad y(t) := t \cdot \sin(4.5t^2 + t + 1) \quad t := -1, -0.99..1.05 \quad T := 0.9$$



- (b) Define functions $slope_{tan}(t)$ and $slope_{norm}(t)$ which, at the point on the curve corresponding to t , would calculate the slopes of the tangent and normal lines, respectively. Evaluate $slope_{tan}(T)$ and $slope_{norm}(T)$.

Solution

$$\text{The derivative of } x(t): \quad xp(t) := \frac{d}{dt}x(t) \qquad \text{The derivative of } y(t): \quad yp(t) := \frac{d}{dt}y(t)$$

$$\text{Slope of the tangent line:} \quad slope_{tan}(t) := \frac{yp(t)}{xp(t)} \qquad slope_{tan}(T) = -1.986$$

$$\text{Slope of the normal line:} \quad slope_{norm}(t) := -\frac{xp(t)}{yp(t)} \qquad slope_{norm}(T) = 0.504$$

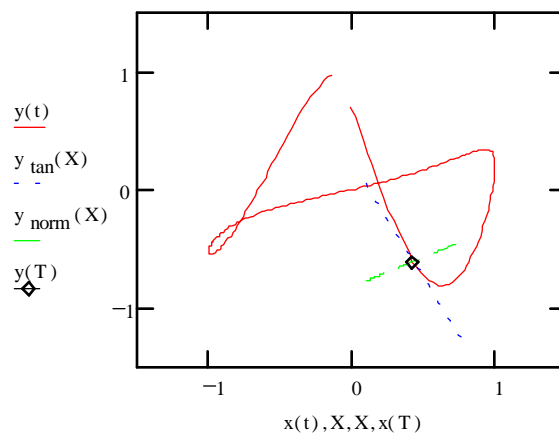
- (c) Define the linear functions $y_{tan}(X)$ and $y_{norm}(X)$ whose graphs are the tangent and the normal lines (respectively) to the curve at the parameter value $T = 0.9$. Copy your graph from part (a), and plot $y_{tan}(X)$ and $y_{norm}(X)$ on it.

Solution

$$\text{Equation of tangent line:} \quad y_{tan}(X) := y(T) + slope_{tan}(T) \cdot (X - x(T))$$

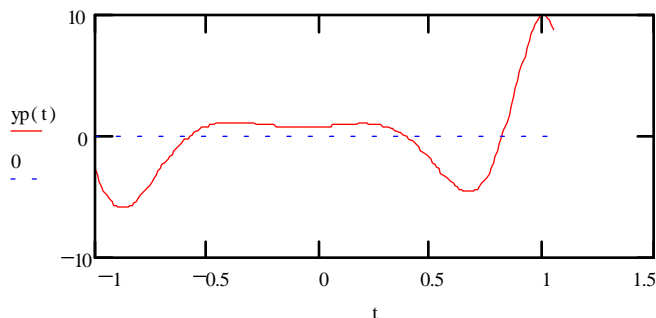
$$\text{Equation of normal line:} \quad y_{norm}(X) := y(T) + slope_{norm}(T) \cdot (X - x(T))$$

$X := 0.1, 0.11..0.8$ (we choose the range variable that covers x values around $x(0.9)=0.427$).



- (d) Use Mathcad to determine all the points of horizontal tangency to the curve, and illustrate one of these points on the graph.

Solution

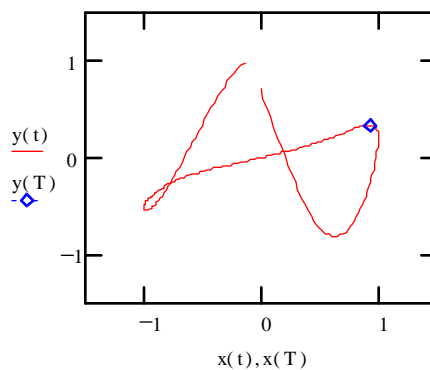


The plot of $yp(t)$ enables us to estimate the location of roots of $yp(t)$:
 $t = -0.5$,
 $t = 0.5$ and
 $t = 0.8$.

Initial guess:	Root calculation:	Parameter value:	Point of horizontal tangency:	
$T := -0.5$	$T1 := \text{root}(yp(T), T)$	$T1 = -0.584$	$x(T1) = -0.983$	$y(T1) = -0.542$
$T := 0.5$	$T2 := \text{root}(yp(T), T)$	$T2 = 0.392$	$x(T2) = 0.923$	$y(T2) = 0.342$
$T := 0.8$	$T3 := \text{root}(yp(T), T)$	$T3 = 0.821$	$x(T3) = 0.627$	$y(T3) = -0.813$

We could numerically check that $xp \neq 0$ at the three t values obtained above. We can also visually verify each of these points appears to be a point of horizontal tangency.

For example $T := T2$



- (e) Above the graph of part (a), start changing the value of T and watch the markers follow the curve. Visually estimate the T values that correspond to the *self-intersection* of the curve in the first quadrant - two distinct estimates are needed!

Use a "Given ... Find" solve block to obtain better approximations for these values.

Solution

$$T1 := 0 \quad T2 := 0.9$$

Given

$$y(T1) = y(T2) \quad x(T1) = x(T2)$$

$$\text{Find}(T1, T2) = \begin{pmatrix} 0.063 \\ 0.984 \end{pmatrix}$$

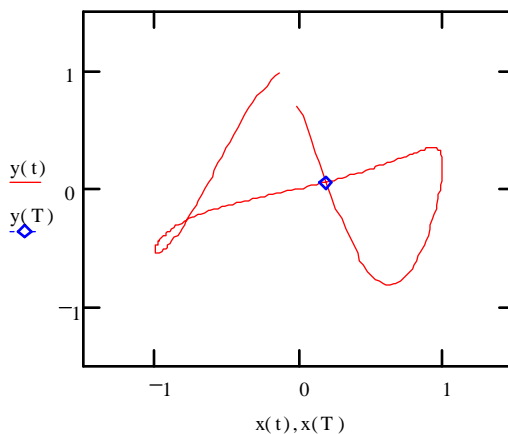
Mathcad responds with values of T1 and T2 that satisfy the constraints.

We verify that the curve passes through the same point at $t=0.063$ and $t=0.984$:

$$x(0.063) = 0.188 \quad y(0.063) = 0.056$$

$$x(0.984) = 0.188 \quad y(0.984) = 0.057$$

$$T := 0.063$$



Problems

1. If a circle of radius a rolls along a straight line, then the curve traced by any given point at a distance b from the center of the circle is called a **trochoid**. Think of the point as located on the spoke of a wheel. If $b < a$, the point is inside the wheel and the trochoid is called a **curtate cycloid**. If $b > a$, imagine that the point is located outside the wheel on a spoke which extends beyond the wheel's radius. In this case, the trochoid is called a **prolate cycloid**. If $b = a$, the trochoid is simply called a **cycloid**.

The parametric equations which describe a trochoid are given by:

$$x(t) = at - b \sin t \quad y(t) = a - b \cos t$$

- (a) In the remainder of this problem, we shall assume $a = 1$. This isn't really a restriction when studying how the shape of a trochoid varies with changing values of a and b .

Why? Briefly explain. Identify the shape corresponding to the special case that is truly different from $a = 1$.

- (b) Use the <Alt>-<Tab> key combination to switch to Program Manager. Double-click on the Maple icon to enter Maple, and then load the file `TROCHOID.MS`. Follow the instructions inside the file. Ask for help if you experience problems. Don't worry about the details of the Maple code at this time. Simply execute the entire file, by selecting **Execute Worksheet** from the **Format** menu, to see the animation that produces a cycloid:

$$x(t) = t - \sin t \quad y(t) = 1 - \cos(t)$$

Note that once the animation window opens, you will have to "press" the "PLAY" button to start the animation (see Comment 4).

After you view the animation, select **Copy** from the **Edit** menu in the animation window. Now use the <Alt>-<Tab> key combination to go back to Mathcad and paste the plot into your document.

- (c) Use <Alt>-<Tab> to return to Maple. In the `TROCHOID.MS` document (which should still be loaded), change the value of b to 0.6 and view the new animation. Copy the plot into your Mathcad document, following the steps described in part (b).
- (d) Repeat part (c), this time changing the value of b to 2. Exit from Maple.
- (e) As b decreases toward zero, what do you expect will happen to the shape of the trochoid? What do you think will happen to the trochoid as b becomes much larger than 1?
- (f) To confirm your expectations in part (e), on separate plots use Mathcad to graph the trochoids corresponding to $b = 10, 5, 2, 1.3, 1, 0.6, 0.2$ and 0 on the interval $[0, 4\pi]$.
2. If a circle of radius b rolls on the inside of a larger circle of radius a , then the curve traced by any given point on the smaller circle is called a **hypocycloid**. The parametric equations which describe the hypocycloid are:

$$x(t) = (a - b) \cos t + b \cos\left(\frac{a - b}{b} t\right) \quad y(t) = (a - b) \sin t - b \sin\left(\frac{a - b}{b} t\right)$$

where t is the central angle of the bigger circle, through which the center of the smaller circle has travelled. Therefore, one circuit of the smaller circle around the larger circle takes place for t on $[0, 2\pi]$. In questions (a)-(f) below, assume the smaller circle has radius $b = 1$.

- (a) Use <Alt>-<Tab> key combination to switch to Program Manager. Double-click on the Maple icon. to enter Maple. Open file `HYPICYCL.MS`. Ask for help if you are experiencing problems. (Do not worry about the details of the Maple code at this time.) Execute the entire file by selecting **Execute Worksheet** from the **Format** menu. When the animation window opens, "click" on the "PLAY" button to see the animation of a hypocycloid with $a = 5$. Play the animation again, stopping it when the small circle has travelled about "half way" around the large circle. Select **Copy** from the **Edit** menu then close the animation window. (Do not close file `HYPICYCL.MS`). <Alt>-<Tab> back to your Mathcad document and paste the plot into your document.



- (b) Use Mathcad to plot the hypocycloid for $a = 2, 3, 4$ and 5 on the interval $[0, 2\pi]$. Describe what you think the graph will look like if a is any integer larger than 5 . Also, briefly comment on why the hypocycloid degenerates to a line segment when $a = 2$.
- (c) Return to file `HYPOCYCL.MS`, change the value of a to $5/2$, and play the animation. Note that the curve is not closed as it was in part (a). Copy and paste the plot into your Mathcad document. Exit from Maple.
- (d) Use Mathcad to graph the hypocycloid discussed in part (c) on the interval $[0, 2\pi]$. Plot the same hypocycloid on the interval $[0, 4\pi]$ to close the curve. Then, on another plot, let $a = 7/2$ and graph the hypocycloid on the smallest interval that closes the curve (such that over a bigger interval, the curve would begin to trace itself again.) Repeat this for $a = 7/3$.
- (e) Based on the examples in part (d), describe how you would produce a starlike figure, first with eight points, and then with any number of points. Verify your answer by plotting the graph for several fractional values of a .
- (f) Let $a = \sqrt{26}$ and plot the hypocycloid for t on $[0, 2\pi]$. How does the graph in this case compare with the graph for $a = 5$ that you found in parts (a) and (b)? Try plotting the curve over longer and longer intervals. Can the curve be closed? Try plotting the hypocycloid for several other irrational values of a . What can you conclude?
- (g) In parts (a)-(f) we assumed that the smaller circle has radius $b = 1$. Explain why this is not really a restriction? (Hint: Define $c = a/b$ in the parametric equations of the hypocycloid.) For what special value of b do the results in parts (a)-(f) not apply? What curve do you get for that special case? (Hint: Visualize the curve that is generated when the smaller circle rolls inside the larger circle, as b approaches that special value.)
3. Consider the motion of a spinning Ferris wheel described by the parametric equations:

$$x1(t) = 20 \sin \frac{\pi t}{20} \quad y1(t) = 20 - 20 \cos \frac{\pi t}{20}$$

and a motion of another wheel with equations

$$x2(t) = 45 + 15 \sin \frac{\pi t}{15} \quad y2(t) = 15 - 15 \cos \frac{\pi t}{15}$$

where we take $0 \leq t \leq 120$.

- (a) Use Mathcad to obtain the graphs of both Ferris wheels on the same plot. Of course the graphs are really circles. You should stretch the plot horizontally so they appear circular.
- (b) Use `<Alt>-<Tab>` to switch to Program Manager and enter Maple. Open file `FERRIS.MS`. Execute the file by selecting **Execute Worksheet** from Maple's **Format** menu. Play and view the animation which shows the location of two points $(x1(t), y1(t))$ and $(x2(t), y2(t))$ on the spinning Ferris wheels starting at $t = 0$ and ending at $t = 120$. Play it again, stopping the animation when the two points appear closest. Locate the



coordinates of these two points by placing the tip of the pointer on them and clicking with the left mouse button. Write down their values for use in part (c) of this problem. Copy and paste the plot into your Mathcad document.

Exit from Maple.

- (c) Use Mathcad to calculate the distance between the two points obtained in part (b).
- (d) The value of the distance obtained in part (c) is only a rough estimate of how close the two points come. Use Mathcad to obtain a more accurate estimate of the minimum distance between the two points.
(Hint: Begin by plotting the distance as a function of time.)
4. (a) Plot the curve described by the parametric equations: $x(t) = 7 + 2 \sinh t$ and $y(t) = -3 + b \cosh t$ for t on the interval $[-2, 2]$ and for $b = 1$. Describe the direction of increasing t along the curve.
- (b) Replot for $b = 0.1, 2$, and 5 . Describe how the shape of the curve changes as the parameter b increases. What happens to the curve when $b = 0$? Describe the direction of increasing t along the graph when $b = 0$.
- (c) By hand, eliminate the parameter t and obtain the Cartesian equation of the curve in terms of b . You should recognize it as the equation of a hyperbola. Type the equation in your document. Now by hand, solve for y in terms of x , and then use Mathcad to graph the upper and lower parts of the hyperbola on the same plot for $b = 1$. Add the asymptotes of the hyperbola to the plot. Explain why only the upper part of the hyperbola was obtained in part (a).
5. Graph the prolate cycloid $x(t) = t - 3 \cos t$, $y(t) = 1 + 3 \sin t$ on the interval $0 \leq t \leq 15$.
- (a) Find the values of the parameter and the Cartesian coordinates of the point where the graph first crosses itself. Follow Example 2e on page 106.
- (b) Determine the area and length of the loop formed by the curve.
- (c) Calculate the equations for both tangent lines at this point and plot them along with the original graph.
6. Graph the curve defined parametrically by $x(t) = \frac{4t}{1+t^3}$ and $y(t) = \frac{4t^2}{1+t^3}$ on the interval $0 < t < 5$.
- (a) What are the Cartesian coordinates of the point on the graph as t tends to infinity.
- (b) Find the points of horizontal tangency to the curve and plot the tangent lines at these points.
- (c) Find the arc length of the loop formed by the curve.

10.2 Activity: Polar Curves

Prerequisites: Read Sections 11.3 - 11.5 LHE.

The objective of this activity is to enhance your understanding of polar curves. You will use Mathcad to plot polar curves and tangent lines to polar curves. You will also determine the length of polar curves and find areas enclosed by polar curves.

Instructions

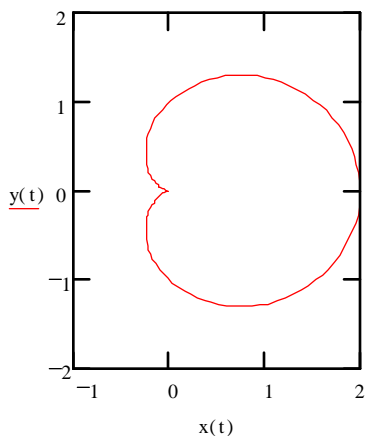
After reading the comments and studying the worked example, open a blank Mathcad document and create your report there. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

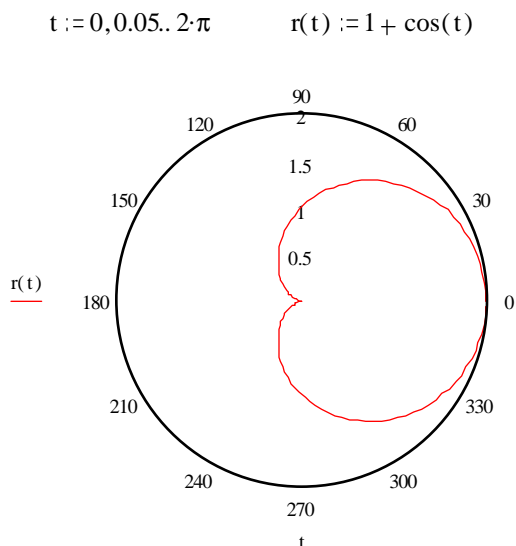
1. Since a polar curve is a special case of a general parametric curve in the plane, we can use Mathcad's X-Y Plot mechanism to generate polar plots in the rectangular coordinates, e.g.,

$$t := 0, 0.05..2 \cdot \pi \quad r(t) := 1 + \cos(t)$$

$$x(t) := r(t) \cdot \cos(t) \quad y(t) := r(t) \cdot \sin(t)$$



Additionally, Mathcad offers a separate polar plot facility, which can be accessed by selecting **Create Polar Plot** from the **Graphics** menu. We demonstrate its use for the cardioid that has been pictured above:



Note that instead of the y and x placeholders, we now have r and θ placeholders. While for the polar curves discussed here r is typically a function of θ , Mathcad's polar plots are not limited to that case.

2. When you create a polar plot in Mathcad, and then select it in a blue frame, Mathcad's **Graphics** menu will turn into the **Polar Plot** menu. Try examining the selections available on that menu - you will find many items that closely parallel those we have already seen in the X-Y Plot case, including **Crosshair** and **Zoom**. However, there are significant differences between the two kinds of plots. For example, there are no placeholders to restrict the θ range - you can only restrict the r range (why?).
3. One important point, related to the previous comment, is that the lower r placeholder **should always be set to zero** - setting it to a nonzero value produces unexpected (and typically undesirable) results.

Example

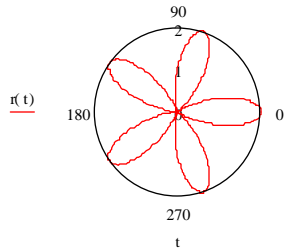
- (a) Create a polar plot for the polar equation $r(\theta) = 1 + \cos 5\theta$ for θ on the interval $[0, 2\pi]$.
- (b) Create the same plot in rectangular coordinates.
- (c) At $\theta = \pi/3$, find the slope of the tangent line and plot it along with the graph.
- (d) Find the total area enclosed by one "petal", and the arc length of one petal.

Solution

(a)

$$t := 0, 0.01.. 2\cdot\pi$$

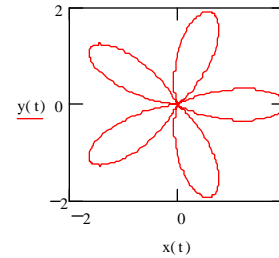
$$r(t) := 1 + \cos(5\cdot t)$$



(b)

$$x(t) := r(t)\cdot\cos(t)$$

$$y(t) := r(t)\cdot\sin(t)$$



Note: We use the parameter t for the polar angle θ .

(c)

$$t := \frac{\pi}{3} \quad m := \frac{r(t)\cdot\cos(t) + \left(\frac{d}{dt}r(t)\right)\cdot\sin(t)}{-r(t)\cdot\sin(t) + \left(\frac{d}{dt}r(t)\right)\cdot\cos(t)}$$

$$m = 5.196$$

m is the slope of the tangent line at $\pi/3$.

$$x_1 := r(t)\cdot\cos(t)$$

$$y_1 := r(t)\cdot\sin(t)$$

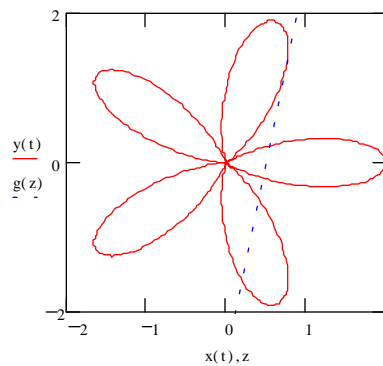
$$z := -2, -1.9.. 2$$

$$g(z) := y_1 + m\cdot(z - x_1)$$

$$t := 0, 0.01.. 2\cdot\pi$$

$$x(t) := r(t)\cdot\cos(t)$$

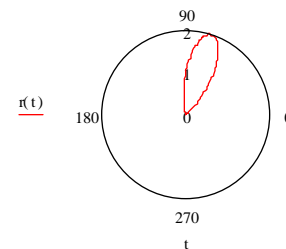
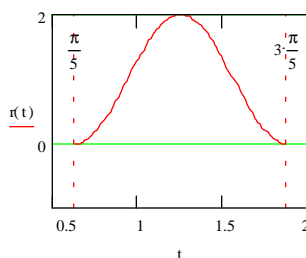
$$y(t) := r(t)\cdot\sin(t)$$



(d) By hand, we find that one petal is given by $\pi/5 < \theta < 3\pi/5$, as shown below.

$$t := \frac{\pi}{5}, \frac{\pi}{5} + 0.01.. \frac{3\cdot\pi}{5}$$

$$r(t) := 1 + \cos(5\cdot t)$$



$$A := \frac{1}{2} \int_{\frac{\pi}{5}}^{3 \cdot \frac{\pi}{5}} r(t)^2 dt \quad A = 0.942 \quad L := \int_{\frac{\pi}{5}}^{3 \cdot \frac{\pi}{5}} \sqrt{\left(\frac{d}{dt}r(t)\right)^2 + r(t)^2} dt \quad L = 4.374$$

Problems

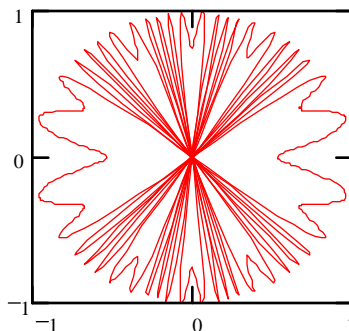
1. Let $r(t) = 1 + b \sin ct$
 - (a) Plot $r(t)$ (use **Polar Plot**) with $b = 1$ and $c = 4$ for $0 < t < 2\pi$. (Use a step size of 0.01 .)
 - (b) Plot dy/dx with respect to t . For efficiency, you should find dr/dt by hand, and use this expression in the equation for dy/dx . Use Mathcad to find all the (x, y) points where the tangent line has a slope of $1/2$. How many are there?
 - (c) Use crosshair and the "root" function to obtain the smallest value of t for which $dy/dx = 1/2$. From this find the x and y coordinates of the point on the curve, and plot the curve and the tangent line. (See Example.)
 - (d) Once you have successfully completed part (c), copy your entire answer to (c) and paste it into your document. Repeat the procedure for all other values of t found in part (b).

2. Let $r(t) = 1 + b \sin(ct)$, with $b = 2$ and $c = 4$. By hand find (exactly) the t intervals for which $r(t)$ creates a small petal and a large petal. Confirm your answer by graphing $r(t)$ on the same plot over these two intervals. Use Mathcad to find the perimeter and area for both a small petal and a large petal.

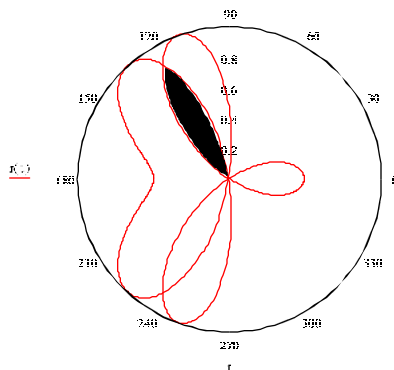
3. Find the area of the common interior of $r = 2 \sin \theta$ and $r = 1 - \cos \theta$.

4. Find the area of the region inside $r = -2 \cos(3\theta)$ and outside $r = 3 + 2 \sin \theta$.

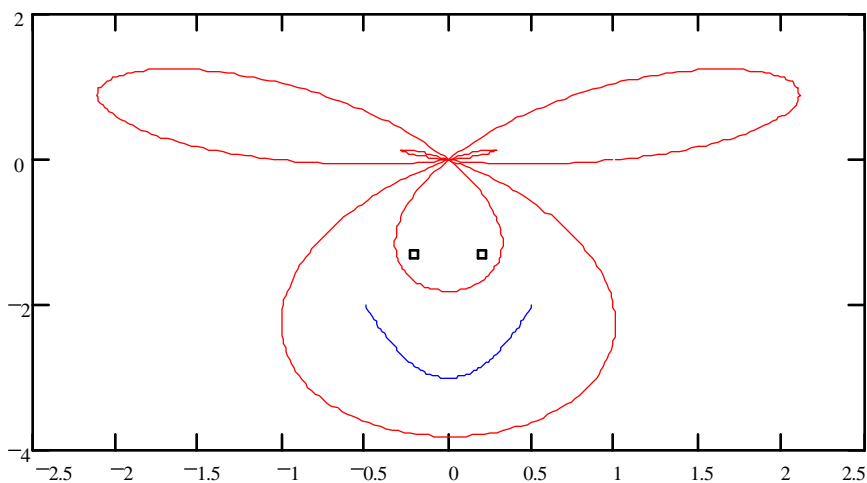
5. (a) Define k to be 0 and $r(t)$ to be $\cos(k \cos(k \cos t))$. Note that $r(t)$ has the form $r(t) = f(\cos t)$ for every value of k . Therefore, its polar graph must be a periodic function of t with period 2π . It follows that the polar graph of $r(t)$ for t on $[0, 2\pi]$ must be a closed curve.
 Plot the polar graph of $r(t)$ for t on $[0, 2\pi]$ using step size 0.01. On another plot, obtain the Cartesian graph of $r(t)$ as a function of t . Explain why the polar graph is a circle.
- (b) Copy and paste your solution to part (a) (definition of k , $r(t)$ and both graphs). In the copy change the value of k to 1, 2, 3, 4, 5, 6, and 7 and observe how the corresponding plots change dramatically.
 For $k = 7$, the polar graph should consist of two large petals and many smaller ones, as shown below. Use Mathcad to find the area of a large petal.



6. Find the perimeter of the region of Problem 3.
7. Find the perimeter of the region of Problem 4.
8. Let $r(t) = 1 + b\sin(ct)$.
 - (a) Taking $b = 3$ and t on the interval $[0, 2\pi]$, obtain the graph of $r(t)$ for $c = 1, 2, 3, 4$, and 5. Describe how the number of petals and their relative orientation appear to depend on integer values of the parameter c .
 - (b) In the plots obtained in part(a), change the value of b to $b = 20$. Describe what happens to the graphs of $r(t)$ for integer values of c when b is large.
 - (c) Based on your answer to part (b), what can you conclude about the graph of $r(t) = b\sin(ct)$ for integer values of c ? Explain.
 - (d) Taking $b = 1/2$ and $c = 5/2$, plot $r(t) = 1 + b\sin(ct)$ on an interval which makes the graph a closed curve. Repeat for $c = 5/3$ and $c = 5/4$. What is the smallest interval required to obtain a closed curve when $c = 5/n$, for any nonzero integer n ? Does this interval change if $c = 6/n$ or $7/n$? Now take $b = 1/2$ and $c = 2.1$, and plot $r(t)$ on the smallest interval required to close the curve.
 - (e) Taking $b = 2$ and $c = 4$, by hand find (exactly) the t intervals over which $r(t)$ creates a small petal and a large petal. Confirm your answers by graphing $r(t)$ on the same plot over these two intervals. Next, use Mathcad to determine the perimeter and area of a small petal and of a large petal.
9. Given $r(\theta) = \cos(\pi \cos \theta + \frac{\pi}{3})$ whose polar plot for $0 \leq \theta \leq 2\pi$ appears below, use Mathcad to approximate the area of the shaded region.



10. Mr. BB's head is described by the polar equation $r = 1 - \sin t + 1.8 \sin 3t$ and t on the interval $[0, 2\pi]$. Find the area and perimeter of his nose, ears, and eyes.



10.3 Homework Help

- After manually sketching the parametric (or polar) curves in the following exercises, use Mathcad to confirm your answer
 - Exercises 3-28 p.674 LHE
 - Exercises 59-64 p.694 LHE
 - Exercises 21-34 p.712 LHE
- As in Example 2 of Section 10.1, several exercises related to tangent lines, vertical/horizontal tangency etc., can be illustrated using Mathcad:

- Exercises 13-28 p.684 LHE

- You can follow the example of Section 10.2 to visualize tangent lines to polar plots, regions within polar curves, etc.
 - Exercises 39-46 p.694 LHE
 - Exercises 3-12, 27-36 p.703 LHE